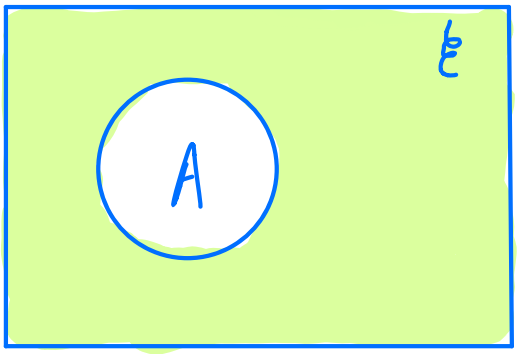
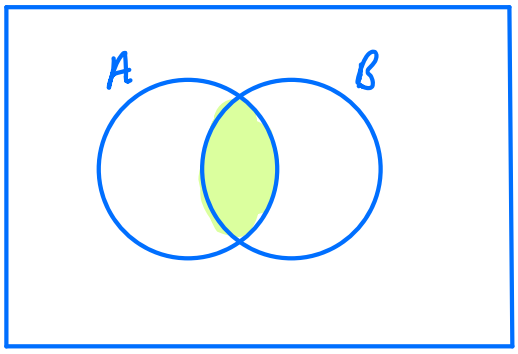
Probability and Venn Diagrams



A'

not A

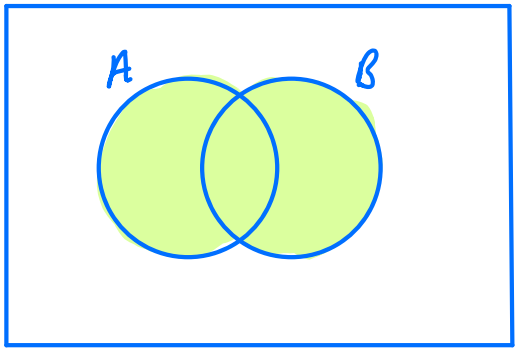


$P(A \cap B)$

Intersection

A and B are independent if and only if

$$P(A \cap B) = P(A) \times P(B)$$



Union

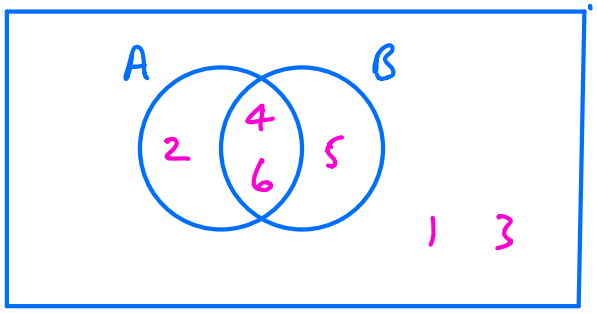
$P(A \cup B)$

$$= P(A) + P(B) - P(A \cap B)$$

Note that if A and B are mutually exclusive (i.e. do not overlap) then $P(A \cup B) = P(A) + P(B)$

Conditional Probability

$$P(A \setminus B) = \frac{P(A \cap B)}{P(B)}$$

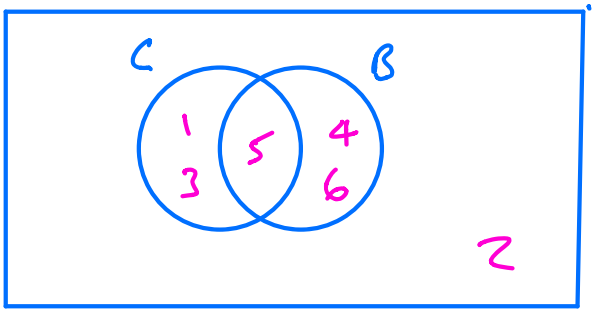


Roll a Dice

A even number $\{2, 4, 6\}$

B number > 3 $\{4, 5, 6\}$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{6} \times \frac{6}{3} = \frac{2}{3}$$



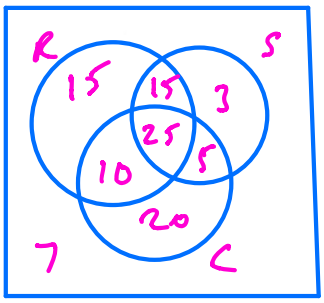
C odd number $\{1, 3, 5\}$

$$P(C \mid B) = \frac{P(C \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{6} \times \frac{6}{3} = \frac{1}{3}$$

JAN 2012 Q6

6. The following shows the results of a survey on the types of exercise taken by a group of 100 people.

65 run
48 swim
60 cycle
40 run and swim
30 swim and cycle
35 run and cycle
25 do all three



(a) Draw a Venn Diagram to represent these data. **(4)**

Find the probability that a randomly selected person from the survey

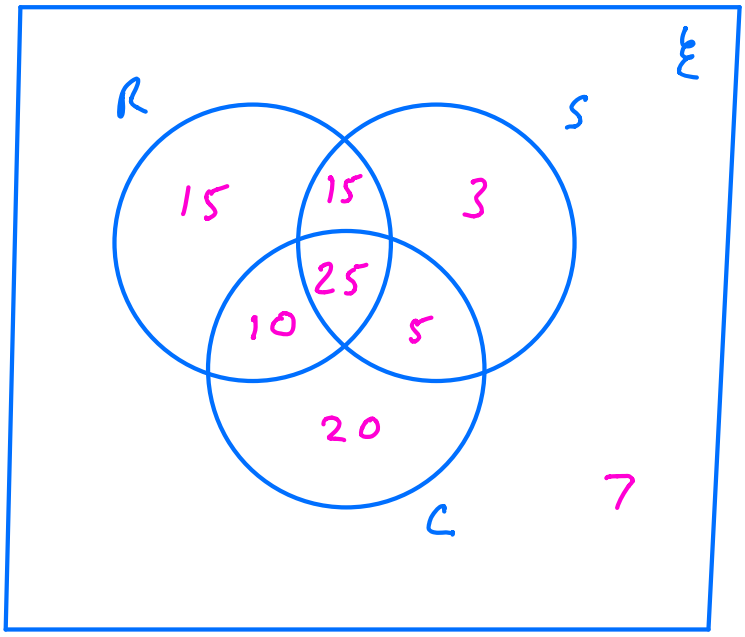
(b) takes none of these types of exercise, **(2)**

(c) swims but does not run, **(2)**

(d) takes at least two of these types of exercise. **(2)**

Jason is one of the above group.
Given that Jason runs,

(e) find the probability that he swims but does not cycle. **(3)**



b) $P(\text{None}) = \frac{7}{100}$

c) $P(S \cap R') = \frac{5+3}{100} = \frac{8}{100}$

d) $P(\text{At least } 2) = \frac{15+10+5+25}{100} = \frac{55}{100}$

e) $P(\text{Swim not cycle} \setminus \text{runs}) = \frac{15}{65}$

Coin	Dice
H 1	T 1
H 2	T 2
H 3	T 3
H 4	T 4
H 5	T 5
H 6	T 6

$P(H \cap 5) = \frac{1}{12}$

$P(H) = \frac{1}{2}$ $\quad$ $P(5) = \frac{1}{6}$

$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

4 Suits H D C S

Red Black

A 2 3 4 .... 10 J Q K

$$P(\text{Diamond}) = \frac{13}{52} = \frac{1}{4}$$

$$P(\text{Queen}) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{Red}) = \frac{26}{52} = \frac{1}{2}$$

$$P(\text{Diamond}) \times P(\text{Queen}) = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52} \checkmark$$

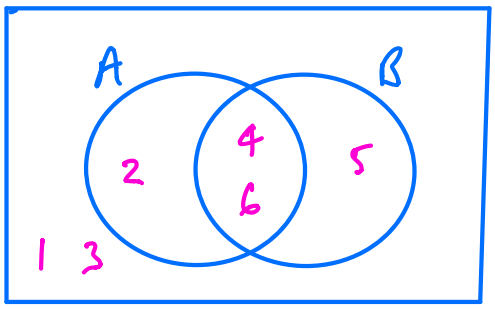
$$P(D \cap Q) = \frac{1}{52} \checkmark \qquad \text{independent}$$

Conditional Probability

Roll a Dice

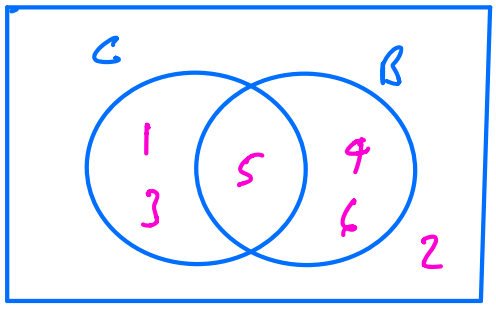
A = even number $= \{2, 4, 6\}$

B = number > 3 $= \{4, 5, 6\}$



$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{6} \times \frac{6}{3} = \frac{2}{3}$$

C = odd number $\{1, 3, 5\}$



$$P(C \mid B) = \frac{P(C \cap B)}{P(B)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{6} \times \frac{6}{3} = \frac{1}{3}$$

Suppose A and B are independent

then $P(A \cap B) = P(A) \times P(B)$

In which case $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

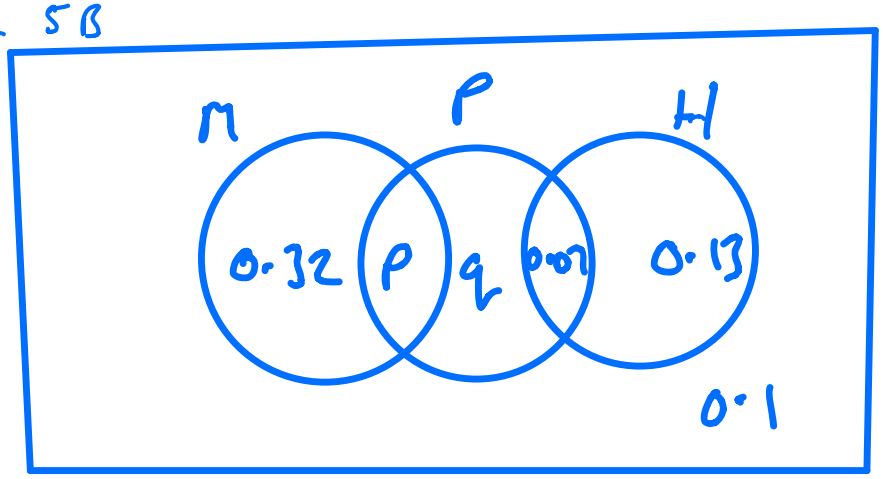
$$= \frac{P(A) \times P(B)}{P(B}$$

$$= P(A)$$

so $P(A)$ is unaffected by the fact B has happened

Exercise 5B

7)



$$P(M) = P(P)$$

$$\Rightarrow \quad 0.32 + p = p + q + 0.07$$

$$0.32 - 0.07 = q$$

$$\underline{q = 0.25}$$

$$0.32 + p + 0.25 + 0.07 + 0.13 + 0.1 = 1$$

$$\underline{p = 0.13}$$