

Integration-Substitution

Exercise 1(e)

$$\begin{aligned}
 1b) \quad & \int \frac{1 + \sin x}{\cos x} dx \\
 &= \int \frac{(1 + \sin x) \cos x dx}{\cos^2 x} \\
 &= \int \frac{(1 + \sin x) \cos x dx}{(1 - \sin^2 x)} \\
 &= \int \frac{1 + u}{1 - u^2} du \\
 &= \int \frac{(1+u)'}{(1+u)(1-u)} du \\
 &= \int \frac{1}{1-u} du \\
 \hline
 &= -\ln|1-u| + C \\
 &= -\ln|1-\sin x| + C
 \end{aligned}$$

Let $v = \sin x$

$$\begin{aligned}
 \frac{dv}{dx} &= \cos x \\
 dv &= \cos x dx
 \end{aligned}$$

$$\int \frac{f'(x)}{f(x)} dx = \ln(f(x))$$

$$1d) \quad \int \frac{z}{\sqrt{z(x-4)}} dx$$

Let $v = \sqrt{x}$

$$\begin{aligned}
 v &= x^{\frac{1}{2}} \\
 \frac{dv}{dx} &= \frac{1}{2} x^{-\frac{1}{2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \int \frac{2}{(v^2-4)} \times 2dx \\
 &= \int \frac{4}{v^2-4} dv \quad \text{Also } dx = v^2 \\
 &= \int \frac{4}{(v+2)(v-2)} dv
 \end{aligned}$$

Aside

$$\frac{4}{(v+2)(v-2)} = \frac{A}{v+2} + \frac{B}{v-2}$$

$$4 = A(v-2) + B(v+2)$$

$$v=2 \quad 4 = 4B \Rightarrow B=1$$

$$v=-2 \quad 4 = -4A \Rightarrow A=-1$$

$$\frac{4}{(v+2)(v-2)} = \frac{1}{v-2} - \frac{1}{v+2}$$

$$= \int \left(\frac{1}{v-2} - \frac{1}{v+2} \right) dv$$

$$= \ln(v-2) - \ln(v+2) + C$$

$$= \ln\left(\frac{v-2}{v+2}\right) + C$$

$$= \ln \left| \frac{\sqrt{x}-2}{\sqrt{x}+2} \right| + C$$

If $\int \sec^4 x dx$

Let $u = \tan x$

$$\frac{du}{dx} = \sec^2 x$$

$$du = \sec^2 x dx$$

$$\int (1 + \tan^2 x) \sec^2 x dx$$

$$= \int (1 + u^2) du$$

$$= u + \frac{u^3}{3} + C$$

$$= \tan x + \frac{\tan^3 x}{3} + C$$

Exercise 1a) 1c) 1e)

$$\int x \sqrt{1+x} dx$$

Let $u = 1+x$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$= \int (u-1) u^{\frac{1}{2}} du$$

Also $x = u-1$

$$= \int (u^{\frac{3}{2}} - u^{\frac{1}{2}}) du$$

$$= \frac{u^{5/2}}{\frac{5}{2}} - \frac{u^{3/2}}{\frac{3}{2}} + C$$

$$= \frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{5} (1+x)^{\frac{5}{2}} - \frac{2}{3} (1+x)^{\frac{3}{2}} + C$$

1c) $\int \sin^3 x dx$

Let $u = \cos x$

$$= \int \sin^2 x \sin x dx$$

$$\frac{du}{dx} = -\sin x$$

$$= \int (1-\cos^2 x) \sin x dx$$

$$du = -\sin x dx$$

$$= \int (1-u^2) du$$

$$-du = \sin x dx$$

$$= \int (u^2 - 1) du$$

$$= \frac{u^3}{3} - u + C$$

$$= \frac{\cos^3 x}{3} - \cos x + C$$

1e) $\int \sec^2 x \tan x \sqrt{1+\tan x} dx$

Let $u^2 = 1+\tan x$

$$= \int (v^2 - 1) v \times 2v \, dv$$

$\frac{2v \, dv}{dx} = \sec^2 x$

$2v \, dv = \sec^2 x \, dx$

$$= \int (2v^4 - 2v^2) \, dv$$

Also $\tan x = v^2 - 1$

$$= \frac{2v^5}{5} - \frac{2v^3}{3} + C$$

$$= \frac{2}{5}(1+\tan x)^{5/2} - \frac{2}{3}(1+\tan x)^{3/2} + C$$

$$2 b) \int_0^2 x(2+x)^3 \, dx \quad \text{Let } u = 2+x$$

$$= \int_2^4 (u-2)u^3 \, du \quad \frac{du}{dx} = 1 \\ du = dx$$

$$\text{Also } x = u-2$$

$$= \int_2^4 (u^4 - 2u^3) \, du \quad \text{Limits } x=2, u=4 \\ x=0, u=2$$

$$= \left[\frac{u^5}{5} - \frac{u^4}{2} \right]_2^4$$

$$= \left(\frac{4^5}{5} - \frac{4^4}{2} \right) - \left(\frac{2^5}{5} - \frac{2^4}{2} \right)$$

$$= 76.8 - 1.6$$

$$= 78.4$$

d) $\int_0^{\frac{\pi}{3}} \sec x \tan x \sqrt{\sec x + 2} dx$

Let $u = \sec x$

Aside $u = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$

$$\frac{du}{dx} = -(\cos x)^{-2}(-\sin x) = \frac{\sin x}{\cos^2 x}$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \sqrt{\sec x + 2} dx$$

$$\frac{du}{dx} = \frac{\sin x}{\cos^2 x}$$

$$du = \frac{\sin x dx}{\cos^2 x}$$

$$\int_1^2 \sqrt{u+2} du$$

Limits

$$\begin{aligned} x &= \frac{\pi}{3} & u &= 2 \\ x &= 0 & u &= 1 \end{aligned}$$

$$\int_3^4 v^{\frac{1}{2}} dv$$

Let $v = u+2$

$$\left[\frac{v^{\frac{3}{2}}}{\frac{3}{2}} \right]_3^4$$

$$\frac{dv}{du} = 1$$

$$dv = du$$

$$\begin{aligned} u &= 2 & v &= 4 \\ u &= 1 & v &= 3 \end{aligned}$$

$$= \frac{2}{3} 4^{\frac{3}{2}} - \frac{2}{3} 3^{\frac{3}{2}}$$

$$= 1.869$$

Hwk Exercise 11E Q2 a, c, e
Q3 a, b, c
Hand in Monday
