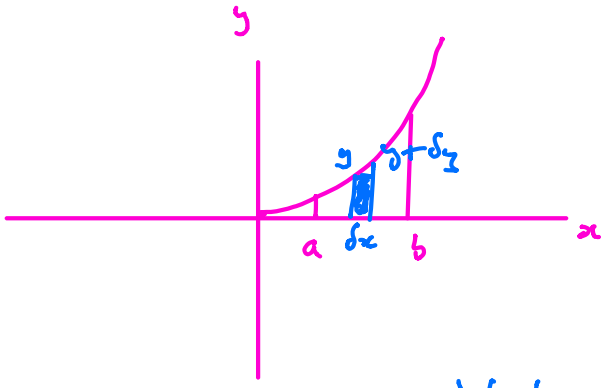
Volumes of Revolution

$y = x^2$



Vol of disc

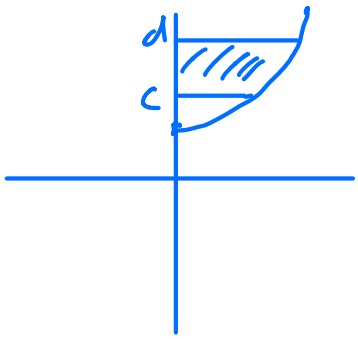
$= \pi y^2 \delta x$

Vol swept out

$\approx \sum \pi y^2 \delta x$

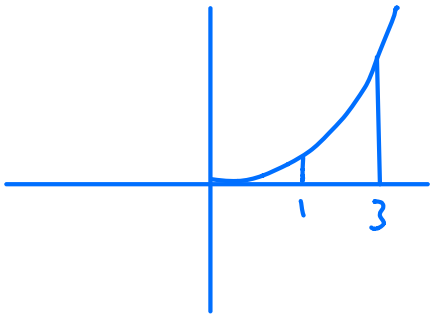
As $\delta x \to 0$ Vol $\to \int_a^b \pi y^2 \, dx$

Spin around y-axis



Vol $\int_c^d \pi x^2 \, dy$

Ex 1



$y = x^2$

Find volume when curve revolved around x-axis from $x = 1$ to $x = 3$

$$\text{Vol} = \int_1^3 \pi y^2 \, dx$$

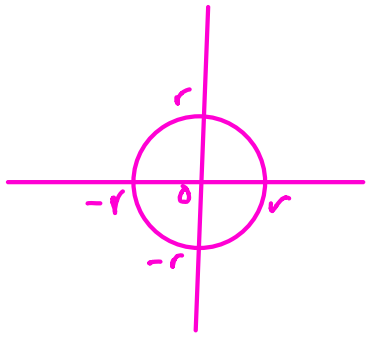
$$= \pi \int_1^3 x^4 \, dx$$

$$= \pi \left[\frac{x^5}{5} \right]_1^3$$

$$= \frac{\pi}{5} \left[243 - 1 \right]$$

$$= \frac{242\pi}{5}$$

Consider $x^2 + y^2 = r^2$



Find vol of hemisphere between $x = 0$ and $x = r$

$$\text{Vol} = \int_0^r \pi y^2 \, dx$$

$$= \pi \int_0^r (r^2 - x^2) \, dx$$

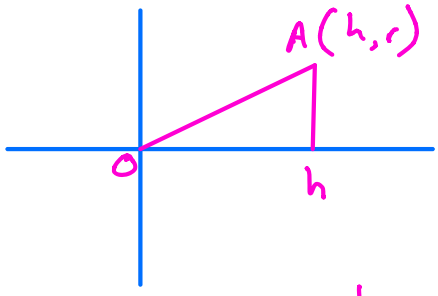
$$= \pi \left[r^2 x - \frac{x^3}{3} \right]_0^r$$

$$= \pi \left[\left(r^3 - \frac{r^3}{3} \right) - (0 - 0) \right]$$

$$= \frac{2}{3} \pi r^3$$

$\therefore$ whole sphere $= 2 \times \frac{2}{3} \pi r^3 = \frac{4}{3} \pi r^3$

Ex 3 Find vol of cone radius r height h



Eqn of OA

$$y = \frac{r}{h}x$$

$$\text{Vol} = \int_0^h \pi y^2 \, dx = \pi \int_0^h \frac{r^2}{h^2} x^2 \, dx$$

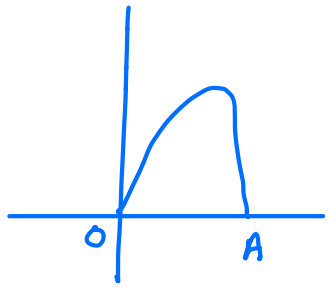
$$= \frac{\pi r^2}{h^2}\left[\frac{x^3}{3}\right]_0^h$$

$$= \frac{\pi r^2}{h^2}\left[\frac{h^3}{3} - 0\right]$$

$$\text{Vol} = \frac{1}{3}\pi r^2 h$$

Exercise 5A

Q5) $y = 9x^{3/2} - 3x^{5/2}$



At A $\quad 9x^{3/2} - 3x^{5/2} = 0$

$$3x^{3/2}(3 - x) = 0 \quad \Rightarrow \quad A(3,0)$$

$$\text{Vol} = \pi \int y^2 \, dx$$

$$y^2 = \left(9x^{3/2} - 3x^{5/2}\right)\left(9x^{3/2} - 3x^{5/2}\right)$$

$$= 81x^3 - 27x^4 - 27x^4 + 9x^5$$

$$\text{Vol} = \pi \int_0^3 \left(81x^3 - 54x^4 + 9x^5\right) dx$$

$$= \pi \left[\frac{81x^4}{4} - \frac{54x^5}{5} + \frac{9x^6}{6} \right]_0^3$$

$$= \frac{2187}{20}\pi$$

Classwork Q 7, Q 8, Q 9