Proof

Exercise 7D

1) $n^2 - n = n(n-1)$

If n is an integer either n or $n-1$ is even

$\therefore$ $n^2 - n$ has a factor of 2 and so is even

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3) Prove $(x+\sqrt{y})(x-\sqrt{y}) \equiv x^2 - y$

$$(x+\sqrt{y})(x-\sqrt{y}) \equiv x^2 + \cancel{x\sqrt{y}} - \cancel{x\sqrt{y}} - y$$

$$\equiv x^2 - y$$

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5) Prove $x^2 + bx \equiv \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

$$\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 \equiv x^2 + 2x\frac{b}{2} + \cancel{\left(\frac{b}{2}\right)^2} - \cancel{\left(\frac{b}{2}\right)^2}$$

$$\equiv x^2 + bx$$

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7) Prove $\left(x - \frac{2}{x}\right)^3 \equiv x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}$

$$\left(x - \frac{2}{x}\right)^2\left(x - \frac{2}{x}\right) \equiv \left(x^2 - 2x.\frac{2}{x} + \frac{4}{x^2}\right)\left(x - \frac{2}{x}\right)$$

$$\equiv \left(x^2 - 4 + \frac{4}{x^2}\right)\left(x - \frac{2}{x}\right)$$

$$\equiv x^3 - 4x + \frac{4}{x}$$

$$\quad - 2x + \frac{8}{x} - \frac{8}{x^3}$$

$$\equiv x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}$$

9) Prove $3n^2 - 4n + 10$ is positive for all n

$$\equiv 3\left[n^2 - \frac{4}{3}n + \frac{10}{3}\right]$$

$$\equiv 3\left[\left(n - \frac{2}{3}\right)^2 + \frac{10}{3} - \frac{4}{9}\right]$$

$$\equiv 3\left(n - \frac{2}{3}\right)^2 + 10 - \frac{4}{3}$$

$$\equiv 3\left(n - \frac{2}{3}\right)^2 + \frac{26}{3} \geqslant \frac{26}{3} \quad \text{since } \left(n - \frac{2}{3}\right)^2 \geqslant 0$$

$\therefore$ $3n^2 - 4n + 10$ is positive for all n

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11) Prove $x^2 + 8x + 20 \geqslant 4$ for all x

$$x^2 + 8x + 20 \equiv (x+4)^2 + 20 - 16$$

$$\equiv (x+4)^2 + 4$$

$$\geqslant 4 \quad \text{since } (x+4)^2 \geqslant 0$$

$\therefore$ $x^2 + 8x + 20 \geqslant 4$ for all x

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13) $px^2 - 5x - 6 = 0$ has 2 distinct roots

$\Rightarrow b^2 > 4ac$

$\Rightarrow (-5)^2 > 4p(-6)$

$25 > -24p$

$24p > -25$

$p > -\frac{25}{24}$

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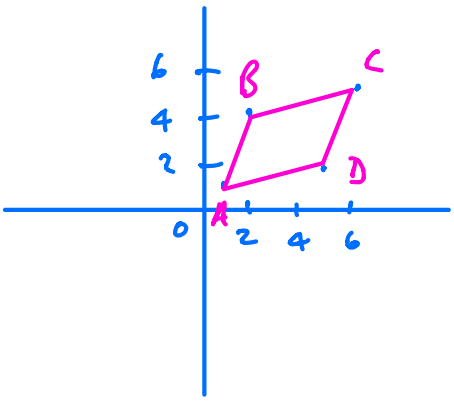
15) Prove $A(1,1)$

$B(2,4)$
$C(6,5)$
$D(5,2)$ form a parallelogram



Gradient AB $= \frac{4-1}{2-1} = \frac{3}{1} = 3$

Gradient CD $= \frac{5-2}{6-5} = \frac{3}{1} = 3$

$\therefore$ AB and CD are parallel

Gradient BC $= \frac{5-4}{6-2} = \frac{1}{4}$

Gradient AD $= \frac{2-1}{5-1} = \frac{1}{4}$

$\therefore$ BC and AD are parallel

Two pairs of parallel sides $\therefore$ ABCD is a parallelogram

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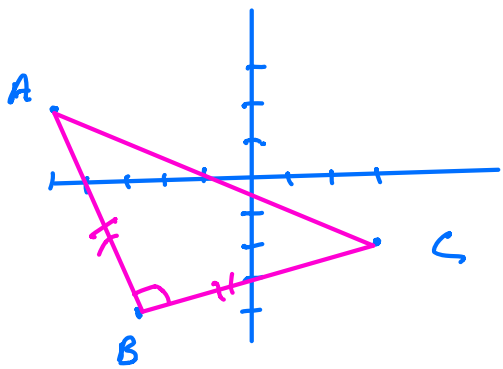
17) $A(-5,2)$
$B(-3,-4)$
$C(3,-2)$



Gradient AB $= \frac{2--4}{-5--3}$

$= \frac{6}{-2} = -3$

Gradient BC $= \frac{-4--2}{-3-3} = \frac{-2}{-6} = +\frac{1}{3}$

$-3 \times \frac{1}{3} = -1$ $\therefore$ AB and BC are $\perp$

$\angle ABC = 90^\circ$

$|AB| = \sqrt{(-5--3)^2 + (2--4)^2} = \sqrt{4+36} = \sqrt{40}$

$|BC| = \sqrt{(3--3)^2 + (-2--4)^2} = \sqrt{36+4} = \sqrt{40}$

$\therefore |AB| = |BC|$

Triangle is right-angled and isosceles

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19) Prove $4y - 3x + 26 = 0$ ①

is a tangent to circle $(x+4)^2 + (y-3)^2 = 100$ ②

From ① $4y = 3x - 26$

$$y = \frac{3x-26}{4}$$

Sub in ②

$$(x+4)^2 + \left(\frac{3x-26}{4} - 3\right)^2 = 100$$

$$(x+4)^2 + \left(\frac{3x-38}{4}\right)^2 = 100$$

$$x^2 + 8x + 16 + \frac{9x^2 - 228x + 1444}{16} = 100$$

$$16x^2 + 128x + 256 + 9x^2 - 228x + 1444 = 1600$$

$$25x^2 - 100x + 100 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

only 1 root $x = 2 \Rightarrow y = -5$

Line and circle intersect only at $(2, -5)$

$\therefore$ line is tangent to circle

Homework Exercise 7D even numbers