Leave blank

1. Solve the equation

$$7\text{sech } x - \tanh x = 5$$

Give your answers in the form $\ln a$ where a is a rational number.

(5)

Leave blank

4. Given that $y = \text{arsinh}(\sqrt{x}), \quad x > 0,$

(a) find $\frac{\mathrm{d}y}{\mathrm{d}x}$, giving your answer as a simplified fraction.

(3)

(b) Hence, or otherwise, find

$$\int_{\frac{1}{4}}^{4} \frac{1}{\sqrt{[x(x+1)]}}\,\mathrm{d}x,$$

giving your answer in the form $\ln\left(\frac{a+b\sqrt{5}}{2}\right)$, where a and b are integers.

(6)

Leave blank

3. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

(b) Solve the equation

$$\cosh 2x - 3\sinh x = 15,$$

giving your answers as exact logarithms.

(5)

Leave blank

5. Given that $y = (\text{arcosh}\, 3x)^2$, where $3x > 1$, show that

(a) $(9x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 36y$,

(5)

(b) $(9x^2 - 1)\frac{d^2y}{dx^2} + 9x\frac{dy}{dx} = 18$.

(4)

Leave blank

Question 5 continued

Leave blank

5. The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3e^{2x}$.

(a) Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

(b) Solve the equation $3\sinh 2x = 13 - 3e^{2x}$, giving your answer in the form $\frac{1}{2}\ln k$, where k is an integer.

(5)

Leave blank

Question 5 continued

Leave blank

5. (a) Differentiate $x \operatorname{arsinh} 2x$ with respect to x.

(3)

(b) Hence, or otherwise, find the exact value of

$$\int_0^{\sqrt{2}} \operatorname{arsinh} 2x \, \mathrm{d}x$$

giving your answer in the form $A \ln B + C$, where A, B and C are real.

(7)

Leave blank

Question 5 continued

Leave blank

7. $f(x) = 5\cosh x - 4\sinh x, \qquad x \in \mathbb{R}$

(a) Show that $f(x) = \frac{1}{2}(e^x + 9e^{-x})$

(2)

Hence

(b) solve $f(x) = 5$

(4)

(c) show that $\int_{\frac{1}{2}\ln 3}^{\ln 3} \frac{1}{5\cosh x - 4\sinh x}\,dx = \frac{\pi}{18}$

(5)

Leave blank

Question 7 continued