Functions Example

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6. The function f is defined by

$$f : x \rightarrow e^{2x} + k^2, \quad x \in \mathbb{R}, \quad k \text{ is a positive constant.}$$

(a) State the range of f. $f(x) > k^2$ **(1)**

(b) Find f^{-1} and state its domain. **(3)**

The function g is defined by

$$g : x \rightarrow \ln(2x), \quad x > 0$$

(c) Solve the equation

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form. **(4)**

(d) Find fg(x), giving your answer in its simplest form. **(2)**

(e) Find, in terms of the constant k, the solution of the equation

$$fg(x) = 2k^2$$

(2)

b) Let $y = e^{2x} + k^2$

swap variables $x = e^{2y} + k^2$

$x - k^2 = e^{2y}$

$\ln(x - k^2) = 2y$

$\frac{1}{2}\ln(x - k^2) = y$

$f^{-1}(x) = \frac{1}{2}\ln(x - k^2)$

Domain $x > k^2$

Leave blank

Question 6 continued

The function g is defined by

$$g : x \rightarrow \ln(2x), \qquad x > 0$$

(c) Solve the equation

$$g(x) + g(x^2) + g(x^3) = 6$$

giving your answer in its simplest form.

(4)

c) $\ln(2x) + \ln(2x^2) + \ln(2x^3) = 6$

$\ln 2 + \ln x + \ln 2 + \ln x^2 + \ln 2 + \ln x^3 = 6$

$3\ln 2 + \ln x + 2\ln x + 3\ln x = 6$

$3\ln 2 + 6\ln x = 6$

$\frac{1}{2}\ln 2 + \ln x = 1$

$\ln\sqrt{2} + \ln x = 1$

$\ln(x\sqrt{2}) = 1$

$e^{\ln(x\sqrt{2})} = e^1 = e$

$x\sqrt{2} = e$

$x = \frac{e}{\sqrt{2}}$

Leave blank

Question 6 continued

(d) Find fg(x), giving your answer in its simplest form.
(2)

(e) Find, in terms of the constant k, the solution of the equation

$$fg(x) = 2k^2$$

(2)

d) $f(x) = e^{2x} + k^2 \qquad g(x) = \ln(2x)$

$fg(x) = f(\ln(2x))$

$= e^{2\ln(2x)} + k^2$

$= e^{\ln 4x^2} + k^2$

$= 4x^2 + k^2$

e) $4x^2 + k^2 = 2k^2$

$4x^2 = k^2$

$2x = k$

$x = \dfrac{k}{2}$