Rational and Irrational Numbers

Definition

A rational number is a number that can be written as $\frac{p}{q}$ where p, q are integers, $q \neq 0$

An irrational number is any number that is not rational $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$

prove rational + irrational = irrational

Let x be irrational

Assume irrational + rational = rational

$$\Rightarrow \quad x + \frac{p}{q} = \frac{h}{k}$$

$$x = \frac{h}{k} - \frac{p}{q} = \frac{hq - pk}{kq}$$

so x is rational contradiction

$\therefore$ irrational + rational = irrational

Types of Decimal Number

Terminating	Infinite Repeating (Recurring)	Any others
eg 0.26	0.123123123 . . .	π

Terminating

0.26 $\qquad$ 0.384 $\qquad$ 0.0512

$\frac{26}{100}$ $\qquad$ $\frac{384}{1000}$ $\qquad$ $\frac{512}{10000}$

So terming decimals are rational

Turning Recurring Decimals Into Fractions

Ex1 $\qquad 0.\dot{3}\dot{7} = 0.37373737\dot{3}\dot{7}$

Let $\quad x = 0.37373\dot{3}\dot{7}$ $\quad$ (1)

$100x = 37.37373\dot{3}\dot{7}$ $\quad$ (2)

(2) − (1) $\qquad 99x = 37$

$x = \frac{37}{99}$

Ex2 $\qquad 0.\dot{4}1\dot{5} = 0.415415\dot{4}1\dot{5}$

Let $\quad x = 0.415\dot{4}1\dot{5}$ $\quad$ (1)

$1000x = 415.415\dot{4}1\dot{5}$ $\quad$ (2)

(2) − (1) $\qquad 999x = 415$

$x = \frac{415}{999}$

Ex 3 $\quad 0.2\dot{7}\dot{4} = 0.27474\dot{7}\dot{4}$

Let $x = 0.27474\dot{7}\dot{4}$

$10x = 2.7474\dot{7}\dot{4}$ ①

$1000x = 274.7474\dot{7}\dot{4}$ ②

② − ① $\quad 990x = 272$

$x = \frac{272}{990} = \frac{136}{495}$

Exercise $\quad$ Turn into fractions

1) $0.\dot{4}\dot{3}$

Let $x = 0.43\dot{4}\dot{3}$

$100x = 43.43\dot{4}\dot{3}$

$99x = 43$ $\quad x = \frac{43}{99}$

2) $0.\dot{2}$

Let $x = 0.22\dot{2}$

$10x = 2.22\dot{2}$

$9x = 2$ $\quad x = \frac{2}{9}$

3) $0.\dot{5}8\dot{3}$

Let $x = 0.583\dot{5}8\dot{3}$

$1000x = 583.583\dot{5}8\dot{3}$ $\quad x = \frac{583}{999}$

$999x = 583$

4) $0.1\dot{2}\dot{3}$

Let $x = 0.123\dot{2}\dot{3}$

$10x = 1.23\dot{2}\dot{3}$

$1000x = 123.23\dot{2}\dot{3}$

$990x = 122$ $\quad x = \frac{122}{990} = \frac{61}{495}$

5) $0.62\dot{5}\dot{5}$

Let $x = 0.625\dot{2}\dot{5}$

$10x = 6.25\dot{2}\dot{5}$

$1000x = 625.252\dot{5}$

$990x = 619$ $\qquad x = \frac{619}{990}$

6) $0.\dot{9}$

Let $x = 0.999\dot{9}$

$10x = 9.999\dot{9}$

$9x = 9$

$x = \frac{9}{9}$ $\qquad x = 1$