Paper 2 Option E

Further Statistics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	H_0: There is no association between language and gender	B1	1.2
		(1)	
(b)	$\frac{54 \times 85}{150} = 30.6$ *	B1*cso	1.1b
		(1)	
(c)	Expected frequencies: Language (French, Spanish, Mandarin); Gender Male: 26.43…, 23.4, 15.16…; Female: 34.56…, [30.6], 19.83…	M1	2.1
	$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(23-26.43)^2}{26.43} + \ldots + \frac{(15-19.83)^2}{19.83}$	M1	1.1b
	Awrt **3.6/3.7**	A1	1.1b
		(3)	
(d)	Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$	M1	3.1b
	As $\sum \frac{(O-E)^2}{E} < 9.210$, the null hypothesis is not rejected	A1	2.2b
		(2)	
(e)	Still not rejected since $\sum \frac{(O-E)^2}{E} < \chi^2_{2,0.1} = 4.605$	B1	2.4
		(1)	
		(8 marks)	

Expected frequencies table for (c):

Expected frequencies		French	Spanish	Mandarin
Gender	Male	26.43…	23.4	15.16…
	Female	34.56…	[30.6]	19.83…

Notes:

(a)
B1: For correct hypothesis in context

(b)
B1*: For a correct calculation leading to the given answer and no errors seen

(c)
M1: For attempt at $\frac{\text{(Row Total)(Column Total)}}{\text{(Grand Total)}}$ to find expected frequencies

M1: For applying $\sum \frac{(O-E)^2}{E}$

A1: awrt 3.6 or 3.7

(d)
M1: For using degrees of freedom to set up a χ^2 model critical value

A1: For correct comparison and conclusion

(e)
A1ft: For correct conclusion with supporting reason

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5\text{E}(X)$	M1	3.1a
	$\text{E}(X) = 1.2$		
	$-1\times c + 0\times a + 1\times a + 2\times b + 3\times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2 \quad \boxed{1}$		
	$\text{P}(Y \geqslant -3) = 0.45$ gives $\text{P}(2 - 5X \geqslant -3) = 0.45$ i.e. $\text{P}(X \leqslant 1) = 0.45$	M1	2.1
	$2a + c = 0.45 \quad \boxed{2}$		
	$2a + b + 2c = 1 \quad \boxed{3}$	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix}\begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. $\boxed{3} - \boxed{2} \Rightarrow b + c = 0.55$ sub. $2(b+c)$ into $\boxed{1} \Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	[$\text{Var}(Y) = 5^2\text{Var}(X)$ implies] $\text{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$\text{P}(Y > X) = \text{P}(2 - 5X > X) \to \text{P}(X < \frac{1}{3})$	M1	3.1a
	$\text{P}(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
		(11 marks)	

Notes:

(a)

M1: For using given information to find an expression for E(X) i.e. use of $\text{E}(Y) = 2 - 5\text{E}(X)$

M1: For use of $\sum x\text{P}(X = x) =$ '1.2'

M1: For use of $\text{P}(Y \geqslant -3) = 0.45$ to set up the argument for solving by forming an equation in a and c

M1: For use of $\sum \text{P}(X = x) = 1$

M1: For solving their 3 linear equations (matrix or elimination)

A1: For any 2 of a, b or c correct

A1: For all 3 correct values

Question 2 notes continued:
Another method for part (a) is: **M1:** For using given information to find the probability distribution for Y leading to an expression for E(Y) **M1:** For use of $\sum y\text{P}(Y = y) = -4$ **M1:** For use of P($Y \geqslant -3$) = 0.45 to set up the argument for solving by forming an equation in a and c **M1:** For use of $\sum \text{P}(Y = y) = 1$ **M1:** For solving their 3 linear equations (matrix or elimination) **A1:** For any 2 of a, b or c correct **A1:** For all 3 correct values
(b) **M1:** For use of Var(Y) = E(Y^2) – [E(Y)]^2 (may be implied by a correct answer) **A1:** For use of Var(aX) = a^2 Var(X) to reach 2.36 or exact equivalent
(c) **M1:** For rearranging to the form P($X < k$) **A1ft:** 0.1' + '025' (provided their a and c and their $a + c$ are all probabilities) **Another method for part (c) is:** **M1:** For comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$ **A1ft:** '0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6)$ $Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= \text{P}(X=2) \times \text{P}(Y=2) = 0.25104\ldots \times 0.21685\ldots$	M1	3.3
	$= 0.05444\ldots$ awrt **<u>0.0544</u>**	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow W \sim \text{Po}(3.8)$	M1	3.4
	$\text{P}(W = 3) = 0.20458\ldots.$ awrt **<u>0.205</u>**	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2)\times 2)$	M1	3.3
	$\text{P}(T < 9) = 0.64819\ldots$ awrt **<u>0.648</u>**	A1	1.1b
		(2)	
(d)	**(i)** Mean = np = **<u>2.4</u>**	B1	1.1b
	(ii) Variance = $np(1-p) = 2.3904$ awrt **<u>2.39</u>**	B1	1.1b
		(2)	
(e)	**(i)** [$D \sim \text{Po}(2.4)$ $\text{P}(D \leqslant 4)$] $= 0.9041\ldots$ awrt **<u>0.904</u>**	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
			(10 marks)

Notes:

(a)

M1: For $\text{P}(X=2) \times \text{P}(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer)

A1: **awrt 0.0544**

(b)

M1: For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer)

A1: **awrt 0.205**

(c)

M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer)

A1: **awrt 0.648**

(d)(i)

B1: For **2.4**

(d)(ii)

B1: For **awrt 2.39**

(e)(i)

B1: For **awrt 0.904**

(e)(ii)

B1: For a correct explanation to support use of Poisson approximation in this case

Question	Scheme	Marks	AOs
4(a)	**(i)** $P(X = 1) = 0.34523\ldots$ awrt **<u>0.345</u>**	B1	1.1b
	(ii) $P(X \leqslant 4) = 0.98575\ldots$ awrt **<u>0.986</u>**	B1	1.1b
		(2)	
(b)	$\frac{(0\times10)+1\times16+2\times7+3\times4+4\times2+(5\times0)+6\times1}{40}=1.4*$	B1*cso	1.1b
		(1)	
(c)	$r = 40 \times$ '0.34523…' $s = 40 \times$ '1 – 0.986…'	M1	3.4
	$r =$ **<u>13.81</u>** $s =$ **<u>0.57</u>**	A1ft	1.1b
		(2)	
(d)	H_0: The Poisson distribution is a suitable model H_1: The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2=\sum\frac{(O-E)^2}{E}=\frac{(10-9.86)^2}{9.86}+\ldots+\frac{(7-(4.51+1.58+0.57))^2}{(4.51+1.58+0.57)}$	M1	1.1b
	awrt **<u>1.1</u>**	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H_0 since $1.10<\chi^2_{2,(0.05)}=5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
		(11 marks)	

Notes:

(a)(i)
B1: awrt 0.345

(a)(ii)
B1: awrt 0.986

(b)
B1*: For a fully correct calculation leading to given answer with no errors seen

(c)
M1: For attempt at r or s (may be implied by correct answers)
A1ft: For both values correct (follow through their answers to part (a))

(d)
B1: For both hypotheses correct (lambda should not be defined so correct use of the model)
M1: For understanding the need to combine cells before calculating the test statistic (may be implied)
M1: For attempt to find the test statistic using $\chi^2=\sum\frac{(O-E)^2}{E}$
A1: awrt 1.1
B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi_2^2(0.05)=5.991$
A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week

Further Mechanics 1 Mark Scheme (Section B)

Question	Scheme	Marks	AOs
5(a)	Using the model and $v^2 = u^2 + 2as$ to find v	M1	3.4
	$v^2 = 2as = 2g \times 2.4 = 4.8g \Rightarrow v = \sqrt{(4.8g)}$	A1	1.1b
	Using the model and $v^2 = u^2 + 2as$ to find u	M1	3.4
	$0^2 = u^2 - 2g \times 0.6 \Rightarrow u = \sqrt{(1.2g)}$	A1	1.1b
	Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR	M1	3.1b
	$e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ *	A1*	1.1b
		(6)	
(b)	Using the model and e = sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$	M1	3.4
	Using the model and $v^2 = u^2 + 2as$	M1	3.4
	$0^2 = 0.25\,(1.2g) - 2gh \Rightarrow h = 0.15$ (m)	A1	1.1b
		(3)	
(c)	Ball continues to bounce with the height of each bounce being a quarter of the previous one	B1	2.2b
		(1)	
		(10 marks)	

Notes:

(a)
M1: For a complete method to find v
A1: For a correct value (may be numerical)
M1: For a complete method to find u
A1: For a correct value (may be numerical)
M1: For finding both v and u and use of Newton's Law of Restitution
A1*: For the given answer

(b)
M1: For use of Newton's Law of Restitution to find rebound speed
M1: For a complete method to find h
A1: For 0.15 (m) oe

(c)
B1: For a clear description including reference to a quarter

Question	Scheme	Marks	AOs
6(a)	Energy Loss = KE Loss – PE Gain	M1	3.3
	$= \frac{1}{2} \times 0.5 \times 25^2 - 0.5g \times 20$	A1	1.1b
	= 58.25 = 58 (J) or 58.3 (J)	A1	1.1b
		(3)	
(b)	Using work-energy principle, $20R = 58.25$	M1	3.3
	$R = 2.9125 = 2.9$ or 2.91	A1ft	1.1b
		(2)	
(c)	Make resistance variable (dependent on speed)	B1	3.5c
		(1)	
			(6 marks)

Notes:

(a)
M1: For a difference in KE and PE
A1: For a correct expression
A1: For either 58 (2sf) or 58.3(3sf)

(b)
M1: For use of work-energy principle
A1ft: For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

(c)
B1: For variable resistance oe

Question	Scheme	Marks	AOs
7(a)	Force = Resistance (since no acceleration) = 30	B1	3.1b
	Power = Force × Speed = 30 × 4	M1	1.1b
	= 120 W	A1 **ft**	1.1b
		(3)	
(b)	Resolving parallel to the slope	M1	3.1b
	$F - 60g\sin\alpha - 30 = 0$	A1	1.1b
	$F = 70$	A1	1.1b
	Power = Force × Speed = 70 × 3	M1	1.1b
	= 210 W	A1 **ft**	1.1b
		(5)	
			(8 marks)

Notes:

(a)
B1: For force = 30 seen
M1: For use of $P = Fv$
A1ft: For 120 (W), follow through on their '30'

(b)
M1: For resolving parallel to the slope with correct no. of terms and $60g$ resolved
A1: For a correct equation
A1: For $F = 70$
M1: For use of $P = Fv$
A1ft: For 210 (W), follow through on their '70'

Question	Scheme	Marks	AOs
8(a)	Use of conservation of momentum	M1	3.1a
	$3mu - 2mu = 3mv + mw$	A1	1.1b
	Use of NLR	M1	3.1a
	$3ue = -v + w$	A1	1.1b
	Using a correct strategy to solve the problem by setting up two equations (need both) in u and v and solving for v	M1	3.1b
	$v = \frac{u}{4}(1 - 3e)$	A1	1.1b
		(6)	
(b)	$\frac{u}{4}(1 - 3e) < 0$	M1	3.1b
	$\frac{1}{3} < e \leq 1$	A1	1.1b
		(2)	
(c)	Solving for w	M1	2.1
	$w = \frac{u}{4}(1 + 9e)$ *	A1 *	1.1b
		(2)	
(d)	Substitute $e = \frac{5}{9}$	M1	1.1b
	$v = -\frac{u}{6}, w = \frac{3u}{2}$	A1	1.1b
	Use NLR for impact with wall, $x = fw$	M1	1.1b
	Further collision if $x > -v$	M1	3.4
	$f\frac{3u}{2} > \frac{u}{6}$	A1	1.1b
	$1 \geq f > \frac{1}{9}$	A1	1.1b
		(6)	
		(16 marks)	

Notes:

(a)
M1: For use of CLM, with correct no. of terms, condone sign errors
A1: For a correct equation
M1: For use of Newton's Law of Restitution, with e on the correct side
A1: For a correct equation
M1: For setting up *two* equations and solving their equations for v
A1: For a correct expression for v

(b)
M1: For use of an appropriate inequality
A1: For a complete range of values of e

(c)
M1: For solving their equations for w
A1: For the given answer

Question 8 notes continued:	
(d)	
M1:	For substituting $e = \frac{5}{9}$ into their v and w
A1:	For correct expressions for v and w
M1:	For use of Newton's Law of Restitution, with e on the correct side
M1:	For use of appropriate inequality
A1:	For a correct inequality
A1:	For a correct range