Algebraic Proof

Usually we are dealing with integers in GCSE Proof

If we need an integer, call it n say

If we need two integers call them m, n say

If we need an even integer let it be

$2n$ where n is an integer

If we need an odd number let it be

$2n+1$ where n is an integer

Ex 1 Prove an even + even = even

Let even numbers be

$2n$ and $2m$ where m, n are integers

$$2n + 2m = 2(n+m)$$

$\therefore$ 2 is factor and so answer is even

Ex 2 Prove an odd × odd = odd

Let odd numbers be

$2n+1$ and $2m+1$

$$(2n+1)(2m+1)$$

$$= 4nm + 2m + 2n + 1$$

$= 2(2nm + m + n) + 1$

This is an even number + 1 so is odd

Ex 3 Prove the sum of any 3 consecutive integers is a multiple of 3

Let integers be n, $n+1$, $n+2$

$$n + n+1 + n+2$$
$$= 3n + 3$$
$$= 3(n+1)$$

3 is a factor so this is a multiple of 3

Exercise

1) Prove odd + odd = even

Let numbers be $2n+1$, $2m+1$

$$= 2n + 1 + 2m + 1$$
$$= 2n + 2m + 2$$
$$= 2(n + m + 1)$$

has a factor of 2 so is even

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2) Prove odd x even = even

Let numbers be $2n+1$ and $2m$

$$2m \times (2n+1)$$
$$= 2(2mn+m)$$

2 is a factor so number is even

Proof

1. The n[th] even number is $2n$.
   a. The next even number can be written as $2n + 2$
   Explain why Even numbers are two units apart so if $2n$ is even next even is $2n+2$
   b. Write down an expression, in terms of n, for the next even number after $2n + 2$. $2n+4$

   c. Show algebraically that the sum of any 3 consecutive even numbers is always a divisible by 6 $2n$, $2n+2$, $2n+4$ as consecutive evens

$$2n + 2n+2 + 2n+4$$
$$= 6n+6$$
$$= 6(n+1)$$

6 is a factor so answer divisible by 6

Squaring Brackets

$$(a+b)^2 = (a+b)(a+b)$$
$$= a^2 + ab + ab + b^2$$

$= a^2 + 2ab + b^2$

= the first term squared + the second term squared + twice the product

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$$(a-b)^2 = (a-b)(a-b)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

same formula but product term is negative

Examples $(x+3)^2 = x^2 + 6x + 9$

$$(2x+1)^2 = 4x^2 + 4x + 1$$
$$(3x-2)^2 = 9x^2 - 12x + 4$$

Exercise

$$(4p+q)^2 = 16p^2 + 8pq + q^2$$
$$(3h+2)^2 = 9h^2 + 12h + 4$$
$$(2x+5)^2 = 4x^2 + 20x + 25$$
$$(x-7)^2 = x^2 - 14x + 49$$
$$(2x-5)^2 = 4x^2 - 20x + 25$$

$$(x-3)^2 = x^2 - 6x + 9$$

$$(y+z)^2 = y^2 + 2yz + z^2$$

$$(4p+1)^2 = 16p^2 + 8p + 1$$

$$(2q+5)^2 = 4q^2 + 20q + 25$$

$$(2n+1)^2 = 4n^2 + 4n + 1$$
