Name: _____

Circular Motion - Horizontal

Date:

Time:

Total marks available:

Total marks achieved: _____

Questions

Q1.

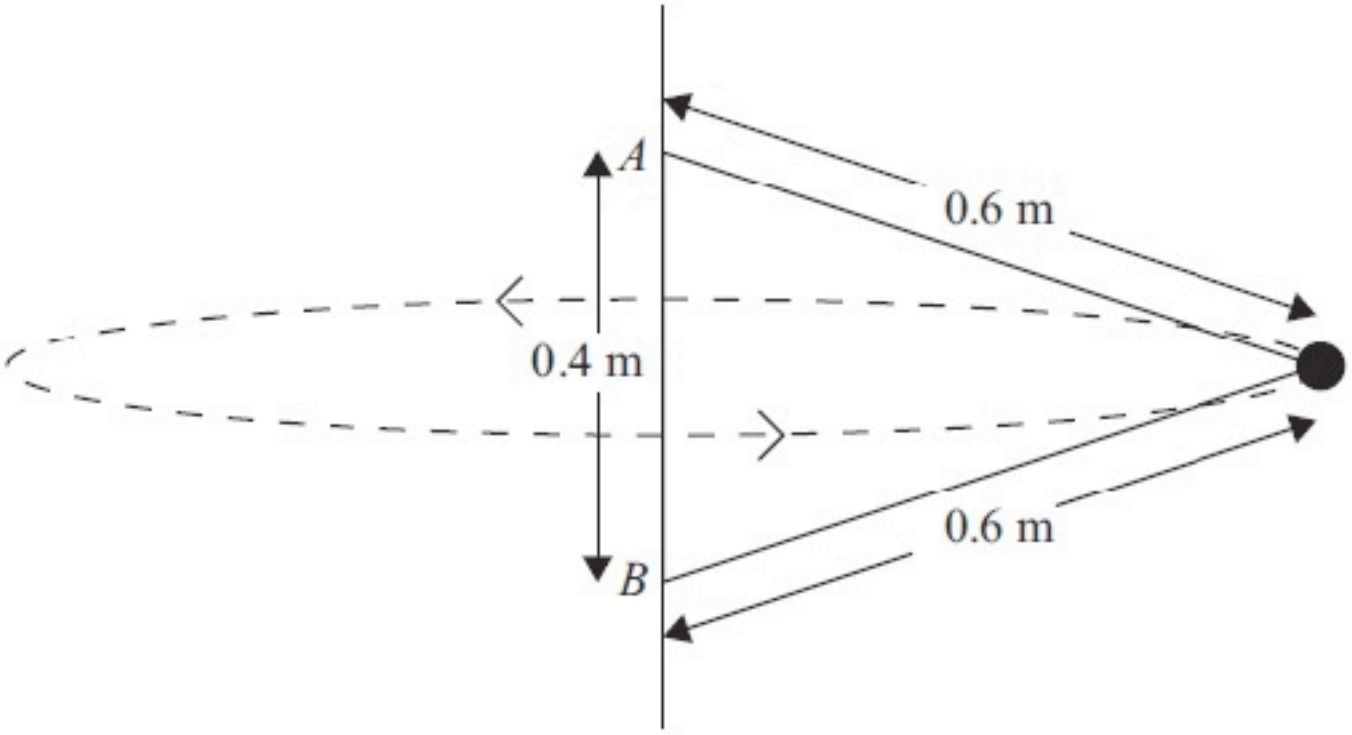

Figure 1

A particle Q of mass 5 kg is attached by two light inextensible strings to two fixed points A and B on a vertical pole. Each string has length 0.6 m and A is 0.4 m vertically above B, as shown in Figure 1.

Both strings are taut and Q is moving in a horizontal circle with constant angular speed 10 rad s^{-1}.

Find the tension in

(i) AQ,
(i) BQ.

(10)
(Total 10 marks)

Q2.

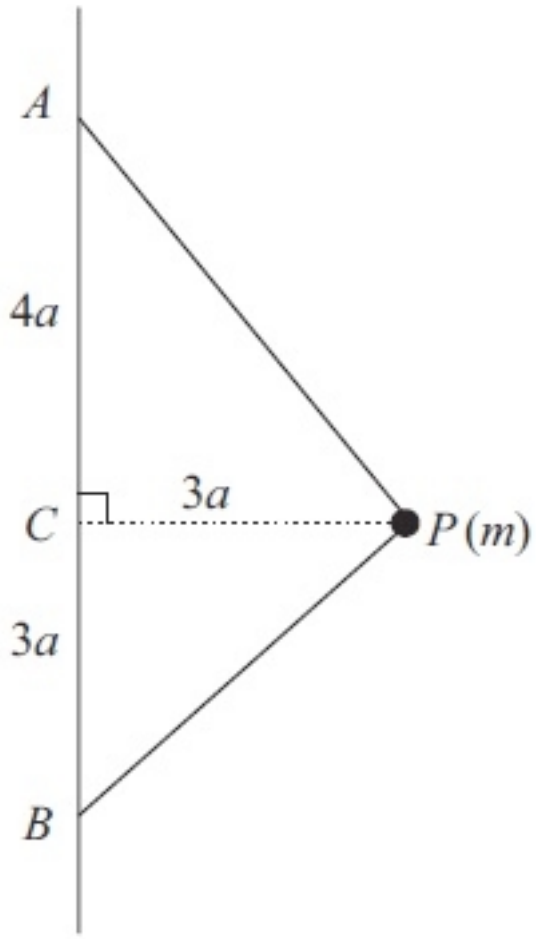

Figure 4

A light inextensible string has its ends attached to two fixed points *A* and *B*. The point *A* is vertically above *B* and *AB* = 7*a*. A particle *P* of mass *m* is fixed to the string and moves in a horizontal circle of radius 3*a* with angular speed ω. The centre of the circle is *C* where *C* lies on *AB* and *AC* = 4*a*, as shown in Figure 4. Both parts of the string are taut.

(a) Show that the tension in *AP* is $\frac{5}{7} m(3a\omega^2 + g)$.

(8)

(b) Find the tension in *BP*.

(2)

(c) Deduce that $\omega \geq \frac{1}{2}\sqrt{\left(\frac{g}{a}\right)}$.

(2)

(Total 12 marks)

Q3.

A rough disc rotates about its centre in a horizontal plane with constant angular speed 80 revolutions per minute. A particle P lies on the disc at a distance 8 cm from the centre of the disc. The coefficient of friction between P and the disc is μ. Given that P remains at rest relative to the disc, find the least possible value of μ.

(7)

(Total 7 marks)

Q4.

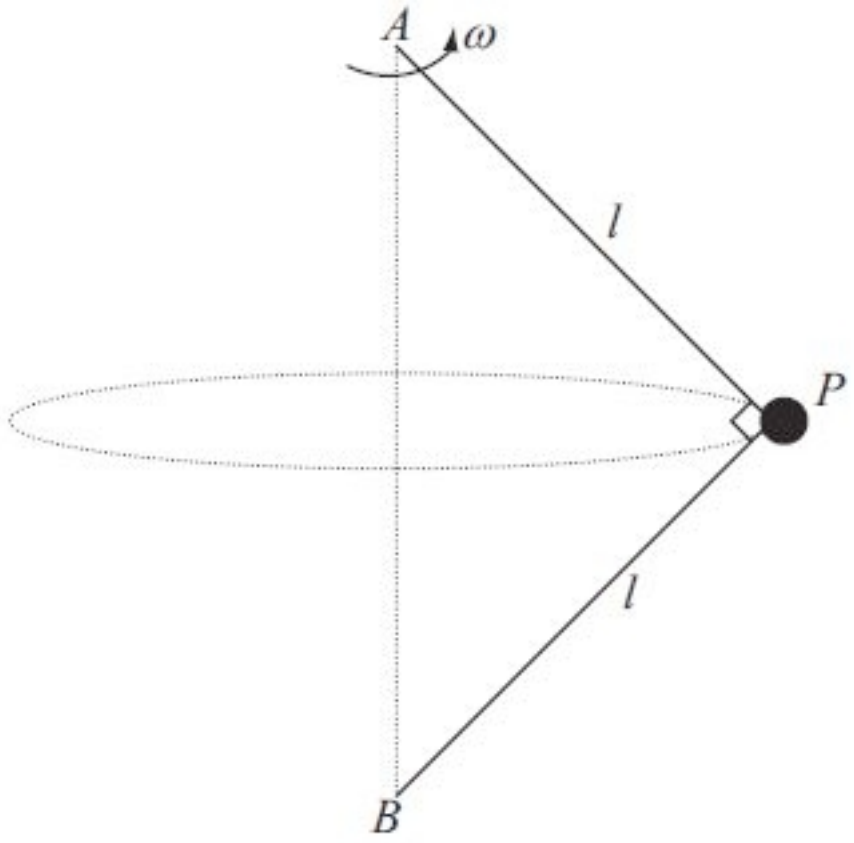

Figure 3

A small ball *P* of mass m is attached to the ends of two light inextensible strings of length *l*. The other ends of the strings are attached to fixed points *A* and *B*, where *A* is vertically above *B*. Both strings are taut and *AP* is perpendicular to *BP* as shown in Figure 3. The system rotates about the line *AB* with constant angular speed ω. The ball moves in a horizontal circle.

(a) Find, in terms of *m*, *g*, *l* and ω, the tension in *AP* and the tension in *BP*.

(8)

(b) Show that

$$\omega^2 > \frac{g\sqrt{2}}{l}.$$

(2)

(Total 10 marks)

Q5.

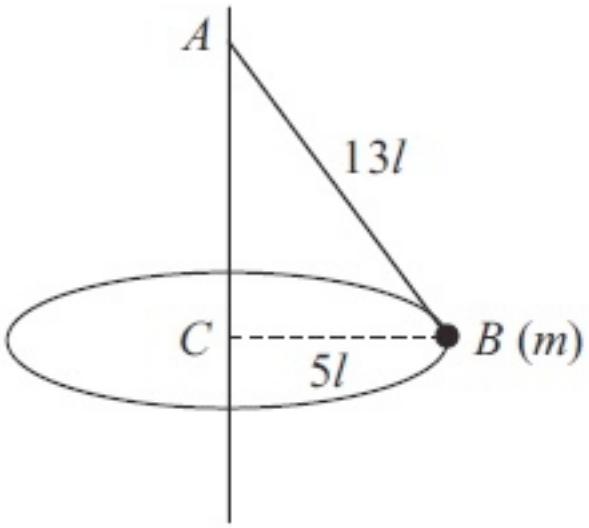

Figure 1

A garden game is played with a small ball *B* of mass *m* attached to one end of a light inextensible string of length 13*l*. The other end of the string is fixed to a point *A* on a vertical pole as shown in Figure 1. The ball is hit and moves with constant speed in a horizontal circle of radius 5*l* and centre *C*, where *C* is vertically below *A*. Modelling the ball as a particle, find

(a) the tension in the string,

(3)

(b) the speed of the ball.

(4)

(Total 7 marks)

Q6.

A bend of a race track is modelled as an arc of a horizontal circle of radius 120 m. The track is not banked at the bend. The maximum speed at which a motorcycle can be ridden round the bend without slipping sideways is 28 m s^{-1}. The motorcycle and its rider are modelled as a particle and air resistance is assumed to be negligible.

(a) Show that the coefficient of friction between the motorcycle and the track is $\frac{2}{3}$.

(6)

The bend is now reconstructed so that the track is banked at an angle α to the horizontal. The maximum speed at which the motorcycle can now be ridden round the bend without slipping sideways is 35 m s^{-1}. The radius of the bend and the coefficient of friction between the motorcycle and the track are unchanged.

(b) Find the value of tan α.

(8)

(Total 14 marks)

Q7.

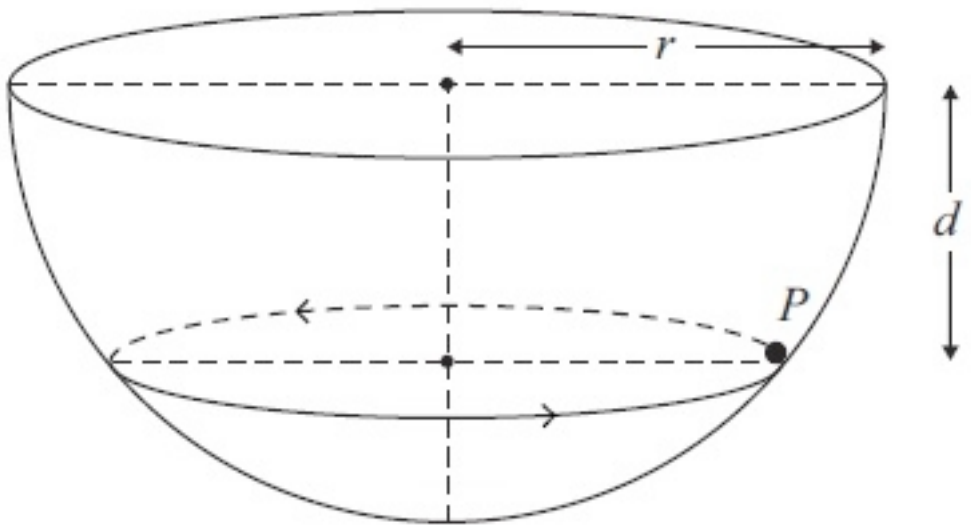

Figure 2

A particle P of mass m moves on the smooth inner surface of a hemispherical bowl of radius r. The bowl is fixed with its rim horizontal as shown in Figure 2. The particle moves with constant angular speed $\sqrt{\left(\frac{3g}{2r}\right)}$ in a horizontal circle at depth d below the centre of the bowl.

(a) Find, in terms of m and g, the magnitude of the normal reaction of the bowl on P.

(4)

(b) Find d in terms of r.

(4)

(Total 8 marks)

Q8.

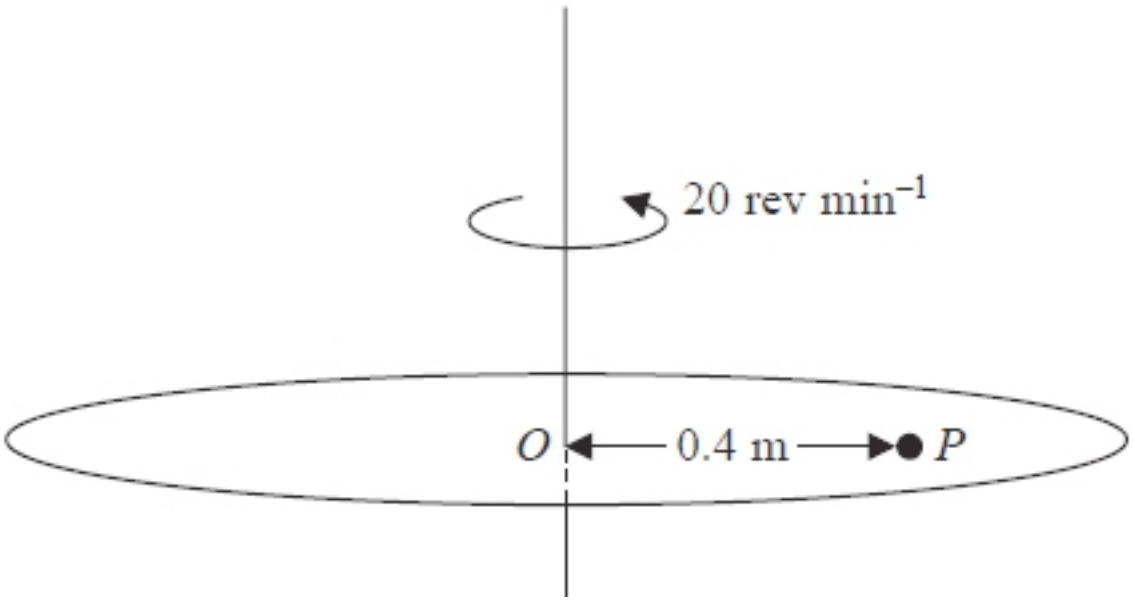

Figure 1

A rough disc is rotating in a horizontal plane with constant angular speed 20 revolutions per minute about a fixed vertical axis through its centre O. A particle P rests on the disc at a distance 0.4 m from O, as shown in Figure 1. The coefficient of friction between P and the disc is μ. The particle P is on the point of slipping.

Find the value of μ.

(6)

(Total 6 marks)

Q9.

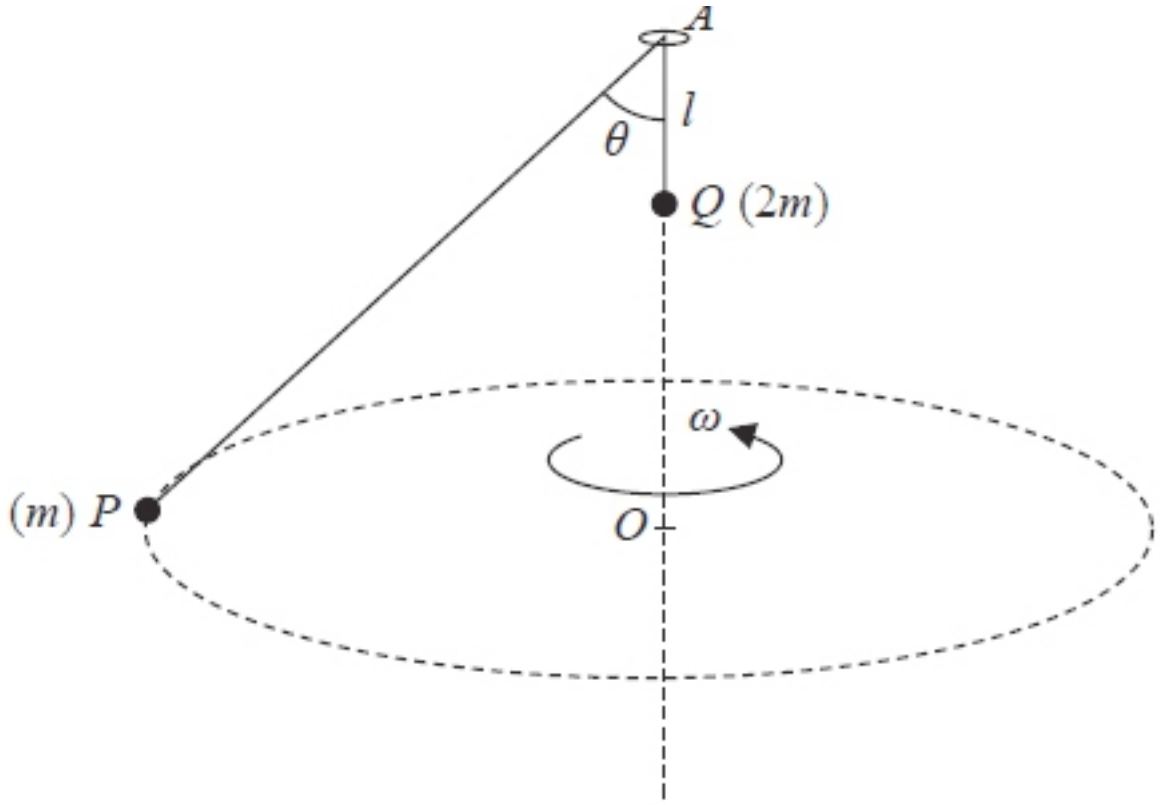

Figure 2

Two particles *P* and *Q*, of mass *m* and 2*m* respectively, are attached to the ends of a light inextensible string of length 6*l*. The string passes through a small smooth fixed ring at the point *A*. The particle *Q* is hanging freely at a distance *l* vertically below *A*. The particle *P* is moving in a horizontal circle with constant angular speed ω. The centre *O* of the circle is vertically below *A*. The particle *Q* does not move and *AP* makes a constant angle θ with the downward vertical, as shown in Figure 2.

Show that

(i) $\theta = 60°$

(i) $\omega = \sqrt{\left(\frac{2g}{5l}\right)}$

(8)

(Total 8 marks)

Q10.

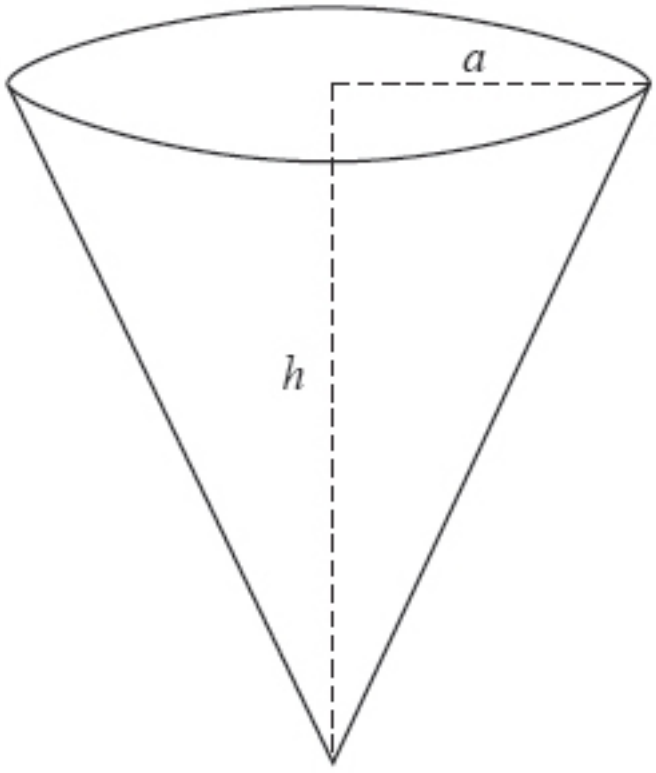

Figure 1

A hollow right circular cone, of base radius *a* and height *h*, is fixed with its axis vertical and vertex downwards, as shown in Figure 1. A particle moves with constant speed *v* in a horizontal circle of radius $\frac{1}{3}a$ on the smooth inner surface of the cone.

Show that $v = \sqrt{\left(\frac{1}{3}hg\right)}$.

(7)

(Total 7 marks)

Q11.

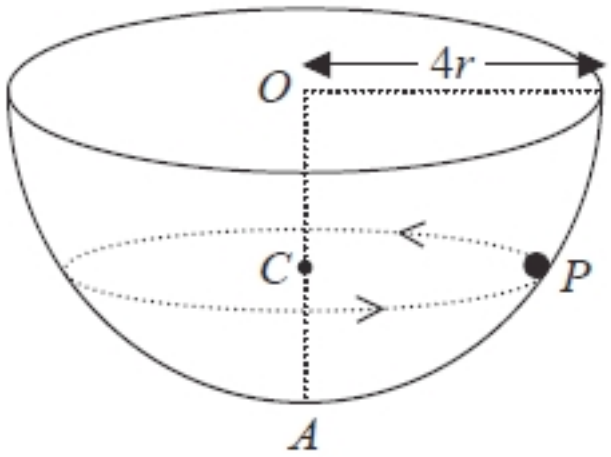

Figure 1

A hemispherical bowl of internal radius $4r$ is fixed with its circular rim horizontal. The centre of the circular rim is O and the point A on the surface of the bowl is vertically below O. A particle P moves in a horizontal circle, with centre C, on the smooth inner surface of the bowl. The particle moves with constant angular speed $\sqrt{\frac{3g}{8r}}$

The point C lies on OA, as shown in Figure 1.

Find, in terms of r, the distance OC.

(9)

(Total 9 marks)

Q12.

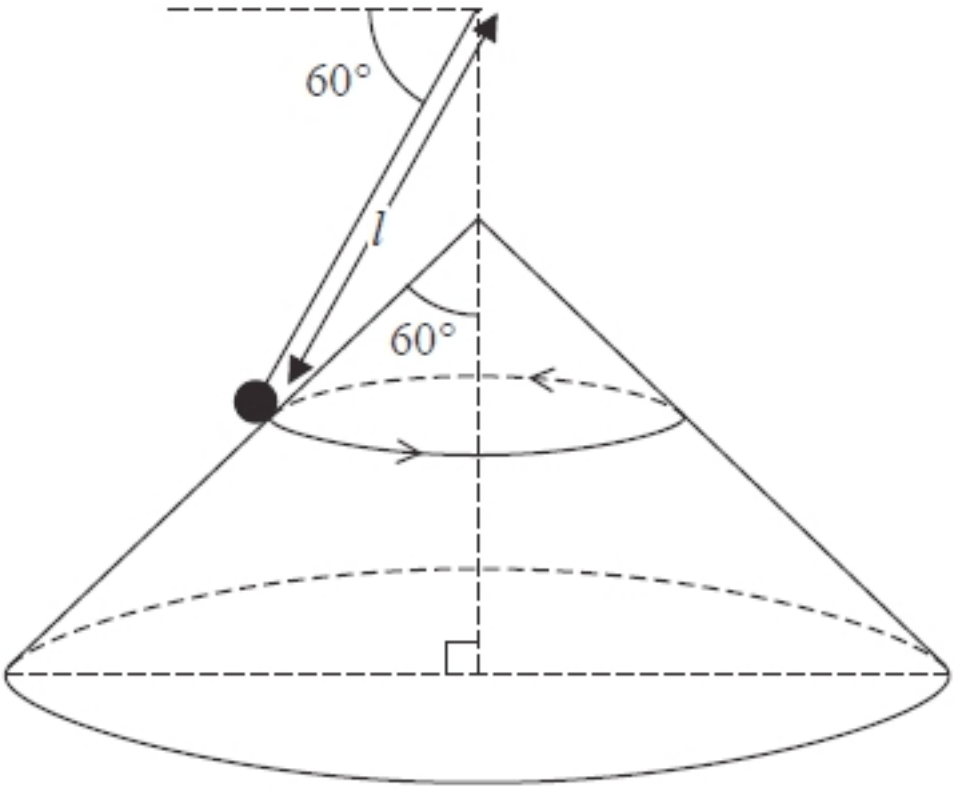

Figure 1

A cone of semi-vertical angle 60° is fixed with its axis vertical and vertex upwards. A particle of mass m is attached to one end of a light inextensible string of length l. The other end of the string is attached to a fixed point vertically above the vertex of the cone. The particle moves in a horizontal circle on the smooth outer surface of the cone with constant angular speed ω, with the string making a constant angle 60° with the horizontal, as shown in Figure 1.

(a) Find the tension in the string, in terms of m, l, ω and g.

(7)

The particle remains on the surface of the cone.

(b) Show that the time for the particle to make one complete revolution is greater than

$$2\pi\sqrt{\frac{l\sqrt{3}}{2g}}$$

(6)

(Total 13 marks)

Q13.

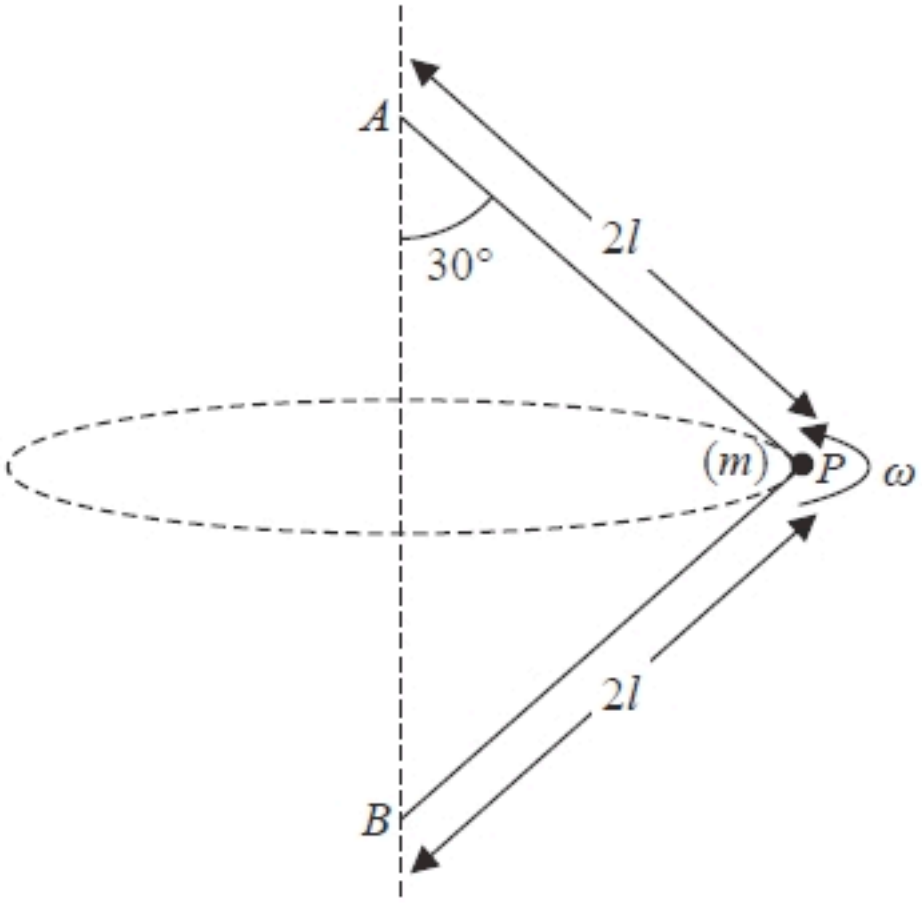

Figure 1

A small ball *P* of mass *m* is attached to the midpoint of a light inextensible string of length 4*l*. The ends of the string are attached to fixed points *A* and *B*, where *A* is vertically above *B*. Both strings are taut and *AP* makes an angle of 30° with *AB*, as shown in Figure 1. The ball is moving in a horizontal circle with constant angular speed ω.

(a) Find, in terms of *m*, *g*, *l* and ω,

(i) the tension in *AP*,
(ii) the tension in *BP*.

(8)

(b) Show that $\omega^2 \geqslant \frac{g\sqrt{3}}{3l}$.

(2)

(Total for question = 10 marks)

Q14.

A car of mass *m* moves in a circular path of radius 75 m round a bend in a road. The maximum speed at which it can move without slipping sideways on the road is 21 m s^{-1}. Given that this section of the road is horizontal,

(a) show that the coefficient of friction between the car and the road is 0.6.

(3)

The car comes to another bend in the road. The car's path now forms an arc of a horizontal circle of radius 44 m. The road is banked at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the car and the road is again 0.6. The car moves at its maximum speed without slipping sideways.

(b) Find, as a multiple of *mg*, the normal reaction between the car and road as the car moves round this bend.

(4)

(c) Find the speed of the car as it goes round this bend.

(5)

(Total 12 marks)

Q15.

A light inextensible string of length *l* has one end attached to a fixed point *A*. The other end is attached to a particle *P* of mass *m*. The particle moves with constant speed *v* in a horizontal circle with the string taut. The centre of the circle is vertically below *A* and the radius of the circle is *r*.

Show that

$$gr^2 = v^2\sqrt{(l^2 - r^2)}.$$

(9)

(Total 9 marks)

Q16.

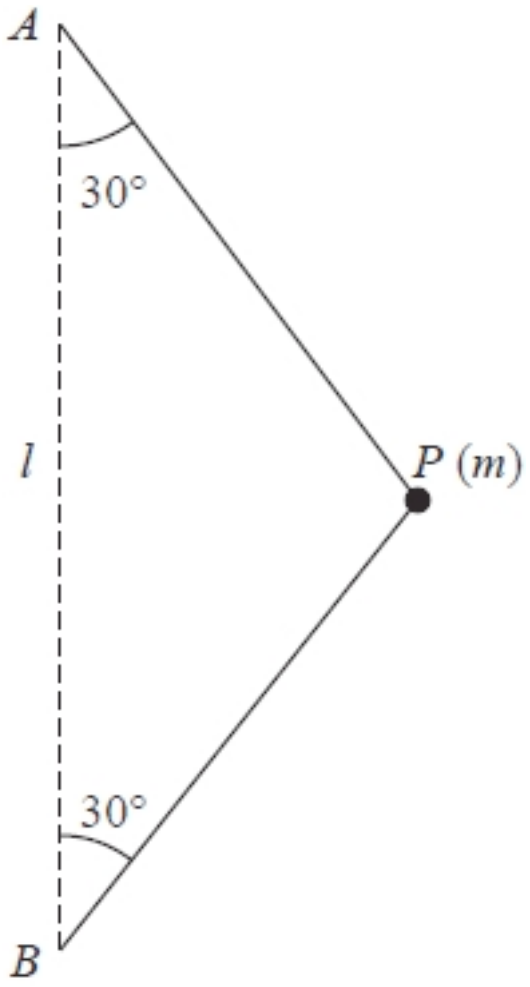

Figure 3

A particle *P* of mass *m* is attached to the ends of two light inextensible strings. The other ends of the strings are attached to fixed points *A* and *B*, where *B* is vertically below *A* and *AB* = *l*. The particle is moving with constant angular speed ω in a horizontal circle. Both strings are taut and inclined at 30° to *AB*, as shown in Figure 3.

(a) (i) Show that the tension in *AP* is $\frac{m\sqrt{3}}{6}(2g + l\omega^2)$

(ii) Find the tension in *BP*.

(9)

(b) Show that the time taken by *P* to complete one revolution is less than $\pi\sqrt{\frac{2l}{g}}$

(4)

(Total for question = 13 marks)

Q17.

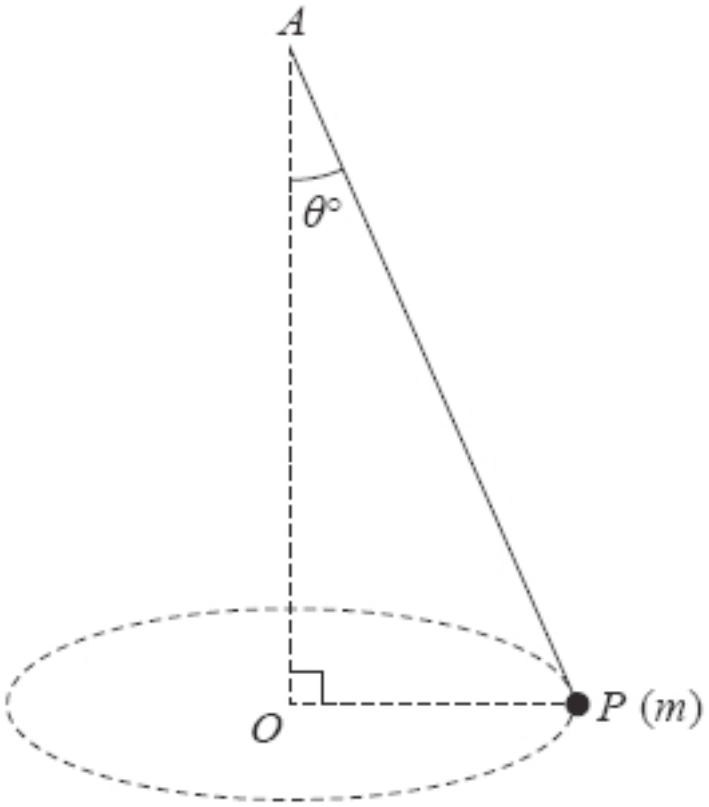

Figure 2

A particle *P* of mass *m* is attached to one end of a light inextensible string. The other end of the string is attached to a fixed point *A*. The particle moves in a horizontal circle with constant angular speed $\sqrt{58.8}$ rad s^{-1}. The centre *O* of the circle is vertically below *A* and the string makes a constant angle $\theta°$ with the downward vertical, as shown in Figure 2.

Given that the tension in the string is 1.2*mg*, find

(i) the value of θ

(ii) the length of the string.

(8)

(Total for question = 8 marks)