Name: _____

Circular Motion - Horizontal

Date:

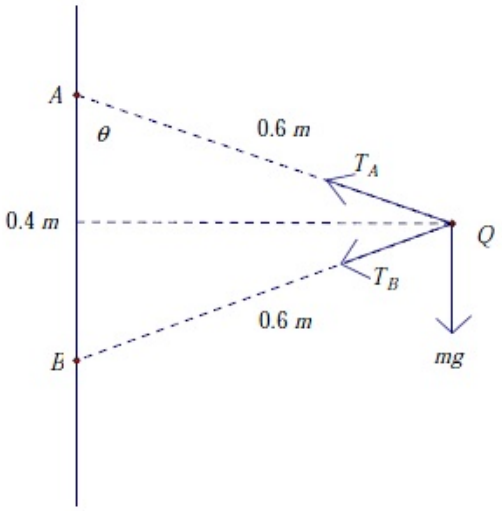
Time:

Total marks available:

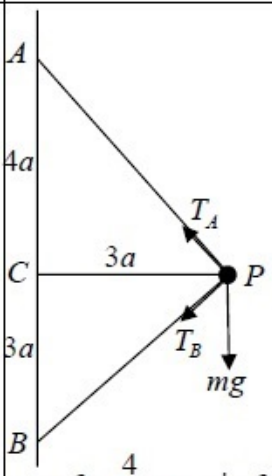
Total marks achieved: _____

Mark Scheme

Q1.

Question Number	Scheme	Marks
		
	$\cos\theta = \frac{0.2}{0.6}\left(=\frac{1}{3}\right)$	B1
	Resolve vertically:	M1
	$T_A\cos\theta = T_B\cos\theta + mg \quad (T_A = T_B + 3mg)$	A2,1,0
	Acceleration towards the centre:	M1
	$T_A\sin\theta + T_B\sin\theta = m\times 0.6\sin\theta\times\omega^2 \quad \left(T_A + T_B = 5\times\frac{3}{5}\times 100 = 300\right)$	A2,1,0
	Substitute values for ω and trig functions and solve to find T_A or T_B	M1
	$T_B + 147 + T_B = 300, \quad 2T_B = 300 - 147 = 153$	A1,A1
	$T_A = 223.5(\text{N}) \quad , \quad T_B = 76.5(\text{N})$	
	$T_A = 224$ or $220 \quad T_B = 76$	
	$T_B = 76.5$ or $77 \quad T_A = 223$	
		(10) **10**

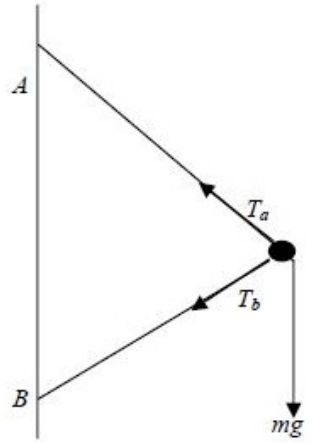
Q2.

Question Number	Scheme	Marks
(a)		
	$\cos\theta = \frac{4}{5}$ or $\sin\theta = \frac{3}{5}$	B1
	R(vert) $T_B \cos 45 + mg = T_A \cos\theta$	M1 A1
	$\frac{1}{\sqrt{2}}T_B + mg = \frac{4}{5}T_A$	
	R(horiz) $T_A \sin\theta + T_B \cos 45 = m \times 3a\omega^2$	M1 A1=A1
	$\frac{3}{5}T_A + \frac{1}{\sqrt{2}}T_B = 3ma\omega^2$	
	$\frac{3}{5}T_A - mg = 3ma\omega^2 - \frac{4}{5}T_a$	M1
	$\frac{7}{5}T_A = 3ma\omega^2 + mg$	
	$T_A = \frac{5}{7}m\left(3a\omega^2 + g\right)$ *	A1 (8)
(b)	$T_b = \sqrt{2}\left(\frac{4}{5}T_a - mg\right)$	M1
	$= \sqrt{2}\left(\frac{4}{7}m\left(3a\omega^2 + g\right) - mg\right)$	
	$= \frac{3\sqrt{2}}{7}m\left(4a\omega^2 - g\right)$ oe	A1
		(2)
(c)	$T_b \geqslant 0 \Rightarrow 4a\omega^2 \geqslant g$	M1
	$\omega^2 \geqslant \frac{g}{4a}$	
	$\omega \geqslant \frac{1}{2}\sqrt{\frac{g}{a}}$ *	A1
	(Allow strict inequalities in (c).)	(2)
		12

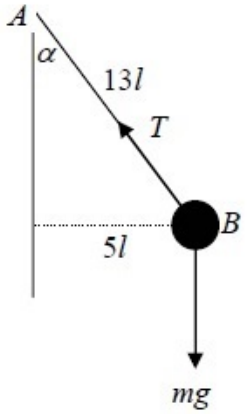
Q3.

Question Number	Scheme	Marks
	$\omega = \frac{80 \times 2\pi}{60}$ rad s^{-1} $\left(= \frac{8\pi}{3} \approx 8.377\ldots\right)$	B1
	Accept $v = \frac{16\pi}{75} \approx 0.67$ ms^{-1} as equivalent	
	$(\uparrow)$ $R = mg$	B1
	For least value of μ $(\leftarrow)$ $\mu mg = mr\omega^2$	M1 A1=A1
	$\mu = \frac{0.08}{9.8} \times \left(\frac{8\pi}{3}\right)^2 \approx 0.57$ accept 0.573	M1 A1 (7)
		[7]

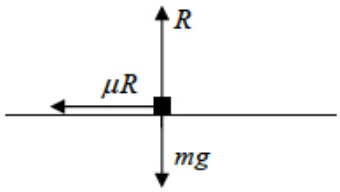
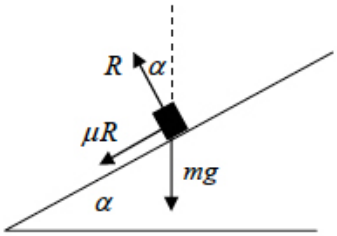
Q4.

Question Number	Scheme	Marks
		
(a)	$r = \frac{l}{\sqrt{2}}$	B1
	$\mathrm{R}(\uparrow) \quad T_a \cos 45 = T_b \cos 45 + mg$	M1A1
	$T_a - T_b = mg\sqrt{2}$ (1)	
	$\mathrm{R}(\rightarrow) \quad T_a \cos 45 + T_b \cos 45 = mr\omega^2$	M1A1
	$T_a \times \frac{1}{\sqrt{2}} + T_b \times \frac{1}{\sqrt{2}} = m\frac{l}{\sqrt{2}}\omega^2$	
	$T_a + T_b = ml\omega^2$ (2)	
	$T_a - T_b = mg\sqrt{2}$ (1)	
	$2T_a = m\left(l\omega^2 + g\sqrt{2}\right)$	M1
	$T_a = \frac{1}{2}m\left(l\omega^2 + g\sqrt{2}\right)$	
	$T_b = ml\omega^2 - T_a$	A1
	$= \frac{1}{2}m\left(l\omega^2 - g\sqrt{2}\right)$	A1
		(8)

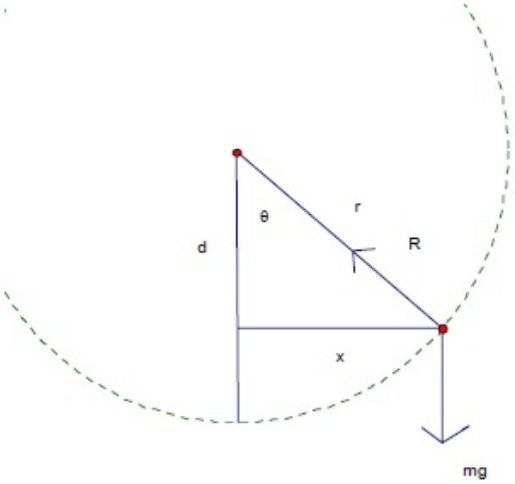
Q5.

Question Number	Scheme	Marks
(a)	 $\cos\alpha = \frac{12}{13}$ $R(\uparrow)\quad T\cos\alpha = mg$ $T \times \frac{12}{13} = mg$ $T = \frac{13}{12}mg$ oe	B1 M1 A1 (3)
(b)	Eqn of motion $T\sin\alpha = m\frac{v^2}{5l}$ $\frac{13mg}{12} \times \frac{5}{13} = m\frac{v^2}{5l}$ $v^2 = \frac{25gl}{12}$ $v = \frac{5}{2}\sqrt{\frac{gl}{3}}$ $\left(\text{accept } 5\sqrt{\frac{gl}{12}} \text{ or } \sqrt{\frac{25gl}{12}} \text{ or any other equiv}\right)$	M1 A1 M1 dep A1 (4) [7]

Q6.

Question Number	Scheme		Marks	
	(a)			
				
	↑ $R = mg$		B1	
	Use of limiting friction, $F_r = \mu R$		B1	
	← $\mu R = \frac{m28^2}{120}$		M1 A1	
	$\mu = \frac{28^2}{120 \times 9.8} = \frac{2}{3}$ ✱	cao	M1 A1	(6)
	(b)			
				
	↑ $R\cos\alpha - \mu R\sin\alpha = mg$		M1 A1	
	← $\mu R\cos\alpha + R\sin\alpha = \frac{mv^2}{r}$		M1 A1	
	$\frac{\mu\cos\alpha + \sin\alpha}{\cos\alpha - \mu\sin\alpha} = \frac{v^2}{rg}$	Eliminating R	M1	
	$\frac{2\cos\alpha + 3\sin\alpha}{3\cos\alpha - 2\sin\alpha} = \frac{25}{24}$	Substituting values	M1	
	leading to $\tan\alpha = \frac{27}{122}$	awrt 0.22	M1 A1	(8)
				[14]

Q7.

Question Number	Scheme	Marks
(a)	 $\leftrightarrow \quad R\sin\theta = mx\omega^2$	M1 A1
	$R \times \frac{x}{r} = mx \times \frac{3g}{2r}$	M1
	$R = \frac{3mg}{2}$	A1
(b)	$\updownarrow \quad R\cos\theta = mg$	M1 A1
	$\frac{3mg}{2} \times \frac{d}{r} = mg$	M1
	$d = \frac{2}{3}r$	A1
		[8]

Q8.

Question Number	Scheme	Marks
	$R(\uparrow)\quad R = mg$	
	$F = \mu mg$	B1
	20 revs per min $= \frac{20}{60} \times 2\pi$ rad s^{-1}	M1A1
	$\left(= \frac{2}{3}\pi \text{ rad s}^{-1}\right)$	
	$R(\rightarrow)\quad \mu mg = m \times 0.4 \times \left(\frac{2}{3}\pi\right)^2$	M1A1ft
	$\mu = \frac{0.4 \times 4\pi^2}{9g}$	
	$\mu = 0.18$ or 0.179	A1
		[6]

Notes for Question

B1 for resolving vertically and using $F = \mu R$ to obtain $F = \mu mg$. This may not be seen explicitly, but give B1 when seen used in an equation.

M1 for attempting to change revs per minute to rad s^{-1}, must see $(2)\pi$. (Can use 60 or 60^2)

A1 for $\frac{20}{60} \times 2\pi$ (rad s^{-1}) oe

M1 for NL2 horizontally along the radius - acceleration in either form for this mark, F or μmg or μm all allowed. r to be 0.4 now or later. This is not dependent on the previous M mark.

A1ft for $\mu mg = m \times 0.4 \times \left(\frac{2}{3}\pi\right)^2$ follow through on their ω

A1cso for $\mu = 0.18$ or 0.179, **must be 2 or 3 sf.**

NB: Use of $\leqslant$: is allowed, provided used correctly, until the final statement, which must be $\mu = \ldots$..

Q9.

Question Number	Scheme	Marks
(i)	For Q $\quad T = 2mg$	B1
	For P $\quad T\cos\theta = mg$	M1
	$\cos\theta = \frac{1}{2} \quad \theta = 60°$ *	A1cso
(ii)	For P $\rightarrow$ $\quad T\sin\theta = mr\omega^2$	M1A1
	$2mg\sin\theta = m \times 5l\sin\theta \times \omega^2$	M1depA1
	$\omega^2 = \frac{2g}{5l} \quad \omega = \sqrt{\frac{2g}{5l}}$ *	A1cso
		[8]

Notes for Question

In this question, award marks as though the question is not divided into two parts - ie give marks for equations wherever seen.

(i)

B1 for using Q (no need to state Q being used) to state that $T = 2mg$ or $T_Q = 2mg$ with $T_P = T_Q$ seen or implied later.

M1 for attempting to resolve vertically for P T must be resolved but sin/cos interchange or omission of g are accuracy errors.
$mg + 2mg = T + T\cos\theta$ gets M0

A1cso for combining the two equations to obtain $\theta = 60°$ *

NB: This is a "show" question, so if no expression is seen for T and just $2mg\cos\theta = mg$ shown, award 0/3 as this equation could have been produced from the required result, so insufficient working.

(ii)

M1 for attempting NL2 for P along the radius. The mass used must be m if the particle is not stated to be P; a mass of $2m$ would imply use of Q. T must be resolved. Acceleration can be in either form.

A1 for $T\sin\theta = mr\omega^2$ or $T\dfrac{\sqrt{3}}{2} = mr\omega^2$

M1 dep for eliminating T between the two equations for P and substituting for r in terms of l and θ dependent on the second but not the first M mark.

A1 for $2mg\sin\theta = m\times 5l\sin\theta\times\omega^2$ or $\dfrac{T\sin\theta}{T\cos\theta} = \tan\theta = 5l\sin\theta\left(\dfrac{\omega^2}{g}\right)$ θ or $60°$

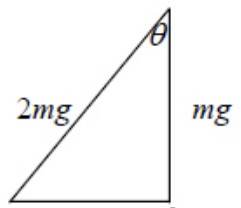
A1cso for re-arranging to obtain $\omega = \sqrt{\dfrac{2g}{5l}}$ * ensure the square root is correctly placed

Alternatives: Some candidates "cancel" the $\sin\theta$ without ever showing it.

M1A1 for $T = m\times 5l\omega^2$
M1A1 for $2mg = 5ml\omega^2$
A1cso as above

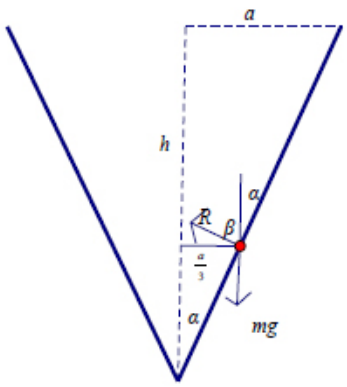
Vector Triangle method: **Triangle must be seen**

$T = 2mg$	B1
$\cos\theta = \dfrac{mg}{2mg}$	M1
$\theta = 60°$	A1
Correct triangle	M1A1
$\sin\theta = \dfrac{5ml\sin\theta\omega^2}{2mg}$	M1A1
$\omega = \ldots$	A1cso (as above)

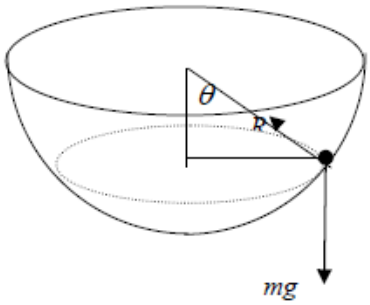


$ma = mr\omega^2$
$= 5ml\sin\theta\,\omega^2$

Q10.

Question Number	Scheme	Marks
		
	Vertical: $R\cos\beta = mg$	M1A1
	Horizontal: $R\sin\beta = \dfrac{mv^2}{r} = \dfrac{3mv^2}{a}$	M1A1
	Divide: $\tan\beta = \dfrac{3mv^2}{amg}$	M1dep
	$\tan\beta = \dfrac{h}{a}$	B1
	$\dfrac{3mv^2}{amg} = \dfrac{h}{a}$, $\dfrac{3v^2}{g} = h$, $v = \sqrt{\dfrac{hg}{3}}$ *AG*	A1
		(7) [7]

Q11.

Question Number	Scheme	Marks
		
	$R\sin\theta = m\times 4r\sin\theta\times\frac{3g}{8r}$	M1A1A1
	$R=\frac{3}{2}mg$	
	$R\cos\theta = mg$	M1A1
	$\frac{3}{2}mg\cos\theta = mg$	M1(dep)
	$\cos\theta=\frac{2}{3}$	A1
	$OC = 4r\cos\theta = 4r\times\frac{2}{3}=\frac{8}{3}r$ oe	M1A1
		[9]

Notes for Question

M1	for NL2 towards C - Accept use of $v=\sqrt{\frac{3g}{8r}}$ and $a=\frac{v^2}{r}$ as a mis-read
A1	for LHS fully correct
A1	for RHS fully correct
ALT:	Work in the direction of R and obtain the same equation with $\sin\theta$ "cancelled". Give M1A1A1 if fully correct, M0 otherwise.
M1	for resolving vertically
A1	for the equation fully correct
M1 dep	for eliminating R between the two equations Dependent on both above M marks
A1	for $\cos\theta=\frac{2}{3}$
M1	for attempting to use trig or Pythagoras to obtain OC
A1 cso	for $OC=\frac{8}{3}r$

Alternative for Question	
M1A1A1	$R\sin\theta = m \times a \times \frac{3g}{8r}$
M1 A1	$R\cos\theta = mg$
M1 A1	$\tan\theta = \frac{3a}{8r}$
M1	$\frac{a}{OC} = \frac{3a}{8r}$
A1	$OC = \frac{8r}{3}$

Q12.

Question Number	Scheme	Marks
(a)	$T\sin 60° + R\sin 60° = mg$	M1 A1
	$T\cos 60° - R\cos 60° = ml\cos 60°\omega^2$	M1 A1 A1
	$T = \frac{1}{2}m(l\omega^2 + \frac{2}{\sqrt{3}}g)$	DM1 A1 (7)
(b)	$R = \frac{1}{2}m(\frac{2}{\sqrt{3}}g - l\omega^2)$	M1 A1
	$\frac{1}{2}m(\frac{2}{\sqrt{3}}g - l\omega^2) > 0$	DM1
	$\omega < \sqrt{\frac{2g}{l\sqrt{3}}}$	A1
	$t > 2\pi\sqrt{\frac{l\sqrt{3}}{2g}}$ **	DM1 A1 (6)
		13

Notes	
(a)	M1 vertical equation A1 correct vertical equation M1 horizontal equation, acceleration in either form A1 correct lhs A1 correct rhs DM1 solve for T A1 correct T
(b)	M1 obtain an expression for R A1 correct expression DM1 setting $R > 0$ A1 correct inequality for w DM1 obtaining an inequality for t A1 correct inequality

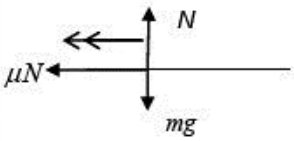
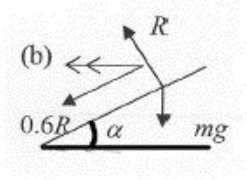
Q13.

Question Number	Scheme	Marks
(a)	R$(\uparrow)$ $T_A \cos 30 = mg + T_B \cos 30$	M1A1
	NL2 $T_A \cos 60 + T_B \cos 60 = mr\omega^2$	M1A1
	$= m \times 2l \cos 60 \omega^2$ or $ml\omega^2$	A1
	$T_A + T_B = 2ml\omega^2$	
	$T_A - T_B = \frac{2mg}{\sqrt{3}}$	
(i)	$T_A = \frac{m}{3}\left(3l\omega^2 + g\sqrt{3}\right)$ oe	DM1A1
(ii)	$T_B = \frac{m}{3}\left(3l\omega^2 - g\sqrt{3}\right)$ oe	A1 (8)
(b)	$T_B \geqslant 0 \Rightarrow 3l\omega^2 \geqslant g\sqrt{3}$	M1
	$\omega^2 \geqslant \frac{g\sqrt{3}}{3l}$ *	A1cso (2)
		[10]

(a) M1 Resolving vertically
A1 Correct equation
M1 NL2 along radius, acceleration in either form
A1 LHS correct
A1 Correct radius substituted and accel in $r\omega^2$. Can be awarded later by implication if work implies correct radius used.
DM1 Solving the two equations to obtain an expression for either tension. Depenent on both previous M marks
A1 Tension in AP correct – simplified to two terms
A1 Tension in BP correct – simplified to two terms

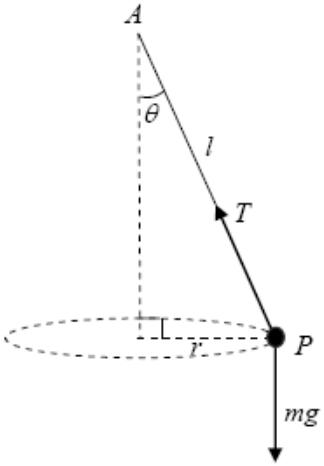
(b) M1 Using their tension in $BP \geqslant 0$ **must be** $\geqslant$ for this mark
A1cso Obtaining the GIVEN answer. Only error allowed is the expression for the tension in AP

Q14.

Question Number	Scheme	Marks
(a)	 $\frac{mv^2}{r} = \mu N, = \mu mg$	M1, A1
	$\mu = \frac{v^2}{rg} = \frac{21^2}{75 \times 9.8} = 0.6$ *	A1 (3)
(b)	 R(↑) $R\cos\alpha, \mp 0.6R\sin\alpha = mg$	M1, A1, A1
	$\Rightarrow R\left(\frac{4}{5} - \frac{3}{5}.\frac{3}{5}\right) = mg \Rightarrow R = \frac{25mg}{11}$	A1 (4)
(c)	R(←) $R\sin\alpha, \pm 0.6R\cos\alpha = \frac{mv^2}{r}$	M1, A1, A1
	$v \approx 32.5 \text{ m s}^{-1}$	dM1 A1cao (5)

	In part (b) M1 needs three terms of which one is mg If cos α and sin α are interchanged in equation this is awarded M1 A0 A1 In part (c) M1 needs three terms of which one is $\frac{mv^2}{r}$ or $mr\omega^2$ If cos α and sin α are interchanged in equation this is also awarded M1 A0 A1 If they resolve along the plane and perpendicular to the plane in part (b), then attempt at $R - mg\cos\alpha = \frac{mv^2}{r}\sin\alpha$, and $0.6R + mg\sin\alpha = \frac{mv^2}{r}\cos\alpha$ and attempt to eliminate v Two correct equations Correct work to solve simultaneous equations Answer In part (c) Substitute R into one of the equations Substitutes into a correct equation (earning accuracy marks in part (b)) Uses $R = \frac{25mg}{11}$ (or $\frac{25mg}{29}$) Obtain $v = 32.5$	M1 A1 A1 A1 (4) M1 A1 A1 M1A1 (5)

Q15.

Question Number	Scheme	Marks
		
	↑ $T\cos\theta = mg$	M1 A1
	← $T\sin\theta = \frac{mv^2}{r}$	M1 A1
	$\tan\theta = \frac{r}{\sqrt{(l^2 - r^2)}}$ or equivalent	M1 A1
	$\tan\theta = \frac{v^2}{rg}$ Eliminating T	M1
	$\frac{r}{\sqrt{(l^2 - r^2)}} = \frac{v^2}{rg}$ Eliminating θ	M1
	$gr^2 = v^2\sqrt{(l^2 - r^2)}$ * cso	A1 **(9)**
		[9]

Q16.

Question Number	Scheme	Marks
(a)	$T_A\cos 30° = mg + T_B\cos 30°$	M1A1
	$T_A - T_B = \frac{2mg}{\sqrt{3}}$	
	Radius $= \frac{1}{2}l\tan 30°$ $\left(= \frac{\sqrt{3}}{6}l \text{ oe}\right)$	B1
	$T_A\cos 60° + T_B\cos 60° = mr\omega^2 = m\left(\frac{1}{2}l\tan 30°\right)\omega^2$	M1A1A1ft
	$T_A + T_B = \frac{ml\omega^2}{\sqrt{3}}$	
(i)	$T_A = \frac{1}{2}\left(\frac{2mg}{\sqrt{3}} + \frac{ml\omega^2}{\sqrt{3}}\right) = \frac{m\sqrt{3}}{6}\left(2g + l\omega^2\right)$ *	DM1A1cso
(ii)	$T_B = \frac{1}{2}\left(\frac{ml\omega^2}{\sqrt{3}} - \frac{2mg}{\sqrt{3}}\right)$ oe	A1cso (9)
(b)	$T_B > 0$ $2mg < ml\omega^2$	M1
	$\omega^2 > \frac{2g}{l}$	A1
	$T = \frac{2\pi}{\omega}$ $T < 2\pi\sqrt{\frac{l}{2g}} = \pi\sqrt{\frac{2l}{g}}$ *	DM1A1cso (4) [13]

(a)M1	Attempt a vertical equation, can have θ for the angle
A1	Completely correct equation, must have numerical angle now
B1	Correct radius seen anywhere
M1	NL2 along the radius. Acceleration in either form and can have r for the radius
A1	Correct sum of tensions (may have a tension on each side)
A1ft	Correct mass x acceleration, follow through their radius
(i)DM1	Solve the equations to either T_A =... or T_B = ... Dependent on both previous M marks. Can be awarded for finding T_A or T_B
A1cso	Correct expression for T_A Given answer so no equivalents allowed.
(ii)A1 cso	Correct expression for T_B. Any equivalent 2 term expression allowed.
Special case	If only one of vertical and radial equations found and the given T_A used to find T_B, award the marks earned for the equation and radius, if used, and B1 for T_B (last A1 in (a) on e-PEN) Max score 5/9
(b)M1	Deducing an inequality from the expression for T_B Can have $2(m)g < (m)l\omega^2$ or $2(m)g \leqslant (m)l\omega^2$ or $2(m)g = (m)l\omega^2$
A1	$\omega^2 > \frac{2g}{l}$ or $\omega^2 \geqslant \frac{2g}{l}$ oe inc equivalent in words.
DM1	Use $T = \frac{2\pi}{\omega}$ with their ω to form an inequality for T, can have $T <$... or $T \leqslant$... Dependent on the first M mark of (b)
A1cso	For a correct final statement from a correct solution. Must be $T <$... or equivalent in words

Q17.

Question Number	Scheme	Marks
(i)	$1.2mg\cos\theta = mg$ or $T\cos\theta = mg$	M1A1
	$\cos\theta° = \frac{1}{1.2}$ $\theta° = \cos^{-1}\frac{1}{1.2}$, $\theta =$ 33.55... (accept 34, 33.6 or better)	A1
(ii)	$1.2mg\sin\theta = mr\omega^2$ or $T\cos\theta = mr\omega^2$	M1A1
	$1.2mg\sin\theta = m\times l\sin\theta\omega^2$	A1
	$1.2mg = 58.8lm \Rightarrow l = \frac{1.2\times 9.8}{58.8} = 0.2(\text{m})$	dM1A1 (8)

M1 Resolve vertically. Tension to be resolved, weight not resolved.

A1 Fully correct equation with substitution for T made.

(i)A1 Correct value of θ Min 2 sf Use of radians scores A0

M1 Attempt NL2 horizontally. Tension must be resolved, acceleration can be in either form.

A1 LHS correct, RHS can be $mr\omega^2$ or $m\frac{v^2}{r}$ here. T substituted now or later

A1 RHS correct, acceleration as shown. $\sin\theta$ may be numerical $\frac{\sqrt{11}}{6}$ or 0.5527... (min 3 sf) or a numerical value for r ($\frac{\sqrt{11}}{30}$ or 0.110...) may be seen.

dM1 Use the above equation to obtain a numerical value for l. Depends on the second M mark

(ii)A1 Correct value of l. Accept 0.2, 0.20, 0.200. Exceptionally allow $\frac{1}{5}$ here.