Product Rule For Differentiation
Let $u$ and $v$ be functions of $x$ Find $\frac{d}{d x}$ (uv)

$$
\begin{equation*}
\text { Let } y=u v \tag{1}
\end{equation*}
$$

For a small change in $x$ say $\delta x$ we will hare a small change in $y, u$ and $v$ say $\delta y$, $\delta u$ and $\delta v$

$$
\begin{equation*}
y+\delta y=(u+\delta u)(v+\delta v) \tag{2}
\end{equation*}
$$

(2)-1)

$$
\begin{aligned}
& \delta_{y}=(u+\delta u)(v+\delta v)-u v \\
& \delta y=y v+v \delta u+u \delta v+\delta u \delta v-y v \\
& \frac{\delta y}{\delta x}=\frac{v \delta u}{\delta x}+u \frac{\delta v}{\delta x}+\delta u \delta v \\
& \delta x
\end{aligned}
$$

$$
\begin{gathered}
\text { Letting }_{\delta x \rightarrow 0} \quad \frac{d y}{d x}=v \frac{d v}{d x}+U \frac{d v}{d x}+O \frac{d v}{d x} \\
\therefore \frac{d y}{d x}=u \frac{d v}{d x}+v \frac{d v}{d x}
\end{gathered}
$$

In plain English, the differential of the product of two functions in $x$ is given by

II
The first times the differential of the second plus the second times the differential of the first,"

Example

$$
\begin{aligned}
\frac{d}{d x} e^{x} \sin x & =e^{x} \cos x+\sin x x e^{x} \\
& =e^{x}(\cos x+\sin x)
\end{aligned}
$$

Quotient Rule For Differentiation
Let $v$ and $v$ be functions of $x$
Find $\frac{d}{d x}\left(\frac{U}{V}\right) \quad$ Leet $y=\frac{U}{V}$
A small change in $x$ say $\delta x$ will cause small changes in $y, u$ and $v$ say $S y$, Jv and $\delta v$

$$
\begin{equation*}
y+\delta y=\frac{u+\delta u}{v+\delta v} \tag{2}
\end{equation*}
$$

(2) - (1)

$$
\begin{aligned}
& y=\frac{u+\delta v}{v+\delta v}-\frac{u}{v} \\
& y=\frac{v(u+\delta u)-u(v+\delta v)}{v(v+\delta v)} \\
& y=\frac{v v+v \delta u-\delta v-u \delta v}{v(v+\delta v)}
\end{aligned}
$$

$$
\begin{aligned}
& y=\frac{v \delta u-u \delta v}{v(v+\delta v)} \\
& \frac{\delta y}{d x}=\frac{V \frac{\delta u}{\delta x}-u \frac{\delta v}{\delta x}}{v(v+\delta v)}
\end{aligned}
$$

Letting $\delta x \rightarrow 0$

$$
\begin{aligned}
& \frac{d y}{d x}=\frac{V \frac{d u}{d x}-u \frac{d v}{d x}}{V(v+0)} \\
& \frac{d y}{d x}=\frac{V \frac{d u}{d x}-\frac{u d v}{d x}}{V^{2}}
\end{aligned}
$$

In plain English. The differential of a quotient is given by:

The bottom times the differential of the top minus the top times the differential of the bottom all over the bottom squared.

Example $\frac{d}{d x} \tan x=\frac{d}{d x} \frac{\sin x}{\cos x}$

$$
\begin{aligned}
& =\frac{\cos x_{x} \cos x-\sin x(-\sin x)}{\cos ^{2} x} \\
& =\frac{\cos ^{2} x+\sin ^{2} x}{\cos ^{2} x} \\
& =\frac{1}{\cos ^{2} x}=\sec ^{2} x
\end{aligned}
$$

Exercise 30
ld) $\frac{d}{d x} 3 x^{5}(5 x-1)^{-1}$

$$
\begin{aligned}
& =3 x^{5} \times(-1)(5 x-1)^{-2}(5)+(5 x-1)^{-1} \times 15 x^{4} \\
& =-15 x^{5}(5 x-1)^{-2}+15 x^{4}(5 x-1)^{-1} \\
& =-15 x^{5}(5 x-1)^{-2}+15 x^{4}(5 x-1)^{-2}(5 x-1) \\
& =15 x^{4}(5 x-1)^{-2}[-x+5 x-1] \\
& =15 x^{4}(5 x-1)^{-2}(4 x-1)
\end{aligned}
$$

