

Homework Review

$$\text{Ex 9 E Q12 } f(x) = y = \frac{2 \cos 2x}{e^{2-x}}$$

$$f'(x) = \frac{e^{2-x} \times (-4 \sin 2x) - 2 \cos 2x (-e^{2-x})}{(e^{2-x})^2}$$

$$\text{At st pt } f'(x) = 0$$

$$\Rightarrow -4 \sin 2x e^{2-x} + 2 \cos 2x e^{2-x} = 0$$

$$2e^{2-x} (\cos 2x - 2 \sin 2x) = 0$$

$$\Rightarrow \cos 2x - 2 \sin 2x = 0$$

$$\cos 2x = 2 \sin 2x$$

$$1 = 2 \frac{\sin 2x}{\cos 2x}$$

$$\frac{1}{2} = \tan 2x$$

$$2x = \tan^{-1} \frac{1}{2}$$

$$2x = 0.4636, \frac{\pi + 0.4636}{}$$

$$x = 1.8026$$

$$f(1.8026) = \frac{2 \cos(1.8026 \pi)}{e^{2-1.8026}} \\ = -1.4684$$

$$f(\pi) = \frac{2 \cos 2\pi}{e^{2-\pi}} = 6.2635$$

$$\text{Range } -1.4684 \leq f(x) < 6.2635$$

Trigonometric Functions and Relationships

Definition $\sec \theta = \frac{1}{\cos \theta}$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

Trig from AS level

$$\cos^2 \theta + \sin^2 \theta = 1 \quad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

A-level relationships

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\therefore \cos^2 \theta$$

$$\frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\therefore \sin^2 \theta$$

$$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\frac{d}{dx} \cot x = \frac{d}{dx} \frac{\cos x}{\sin x}$$

$$\frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$$

$$\frac{-\sin^2 x - \cos^2 x}{\sin^2 x}$$

$$\frac{-1(\sin^2 x + \cos^2 x)}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$$

$$\begin{aligned}
 \frac{d}{dx} \sec x &= \frac{d}{dx} \frac{1}{\cos x} \\
 &= \frac{d}{dx} (\cos x)^{-1} \\
 &= -1 (\cos x)^{-2} (-\sin x) \\
 &= \frac{\sin x}{\cos^2 x} \quad \text{or} \quad \frac{\sin x}{\cos x} \times \frac{1}{\cos x} \\
 &= \tan x \sec x
 \end{aligned}$$

$$\begin{aligned}
 \frac{d}{dx} \csc x &= \frac{d}{dx} \frac{1}{\sin x} = \frac{d}{dx} (\sin x)^{-1} \\
 &= -1 (\sin x)^{-2} \cos x \\
 &= -\frac{\cos x}{\sin^2 x} \\
 \text{or} \quad &- \cot x \csc x
 \end{aligned}$$

Inverse Trig Functions

$$y = \sin^{-1} x = \arcsin x$$

$$y = \cos^{-1} x = \arccos x$$

$$y = \tan^{-1} x = \arctan x$$

Differentiating inverse trig functions

$$y = \sin^{-1} x$$

$$\sin y = x$$

$$\cos y = \frac{dx}{dy}$$

$$\frac{1}{\cos y} = \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-\sin^2 y}} = \frac{dy}{dx}$$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$$

$$\therefore \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$y = \tan^{-1} x$$

$$\tan y = x$$

$$\sec^2 y = \frac{dx}{dy}$$

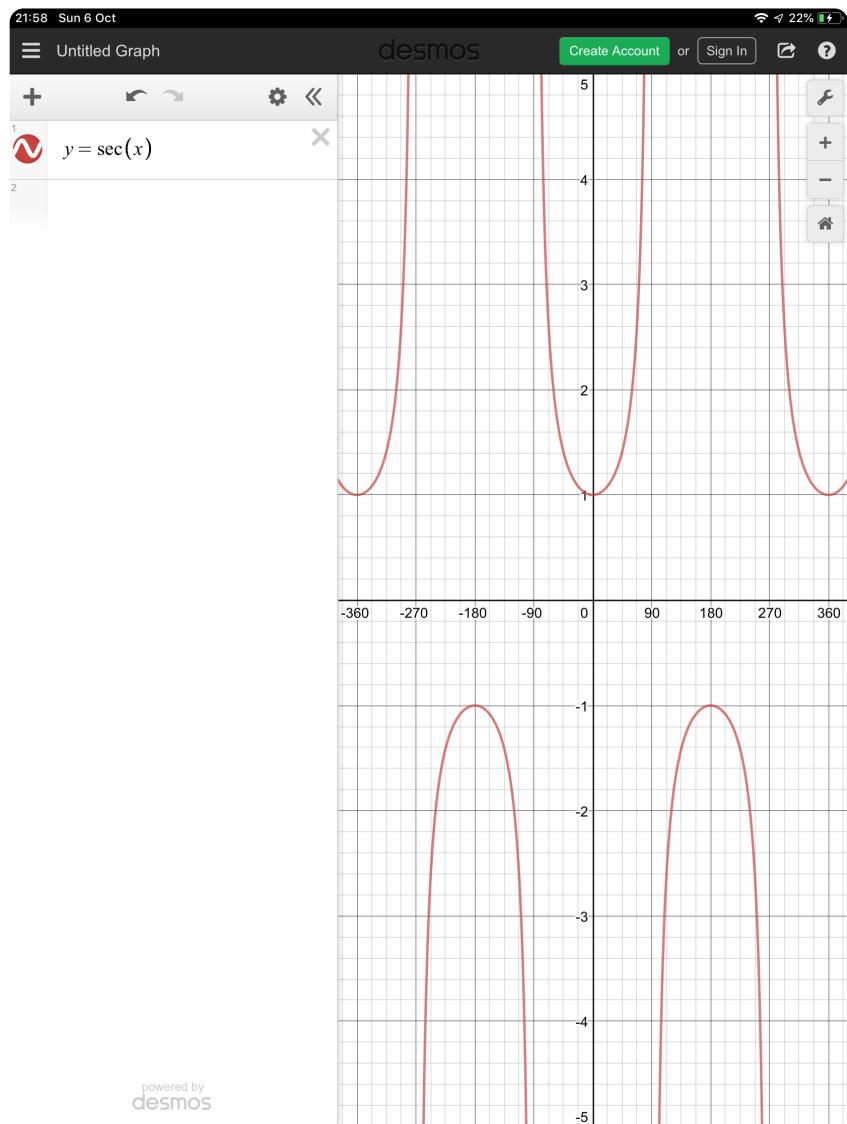
$$1 + \tan^2 y = \frac{dx}{dy}$$

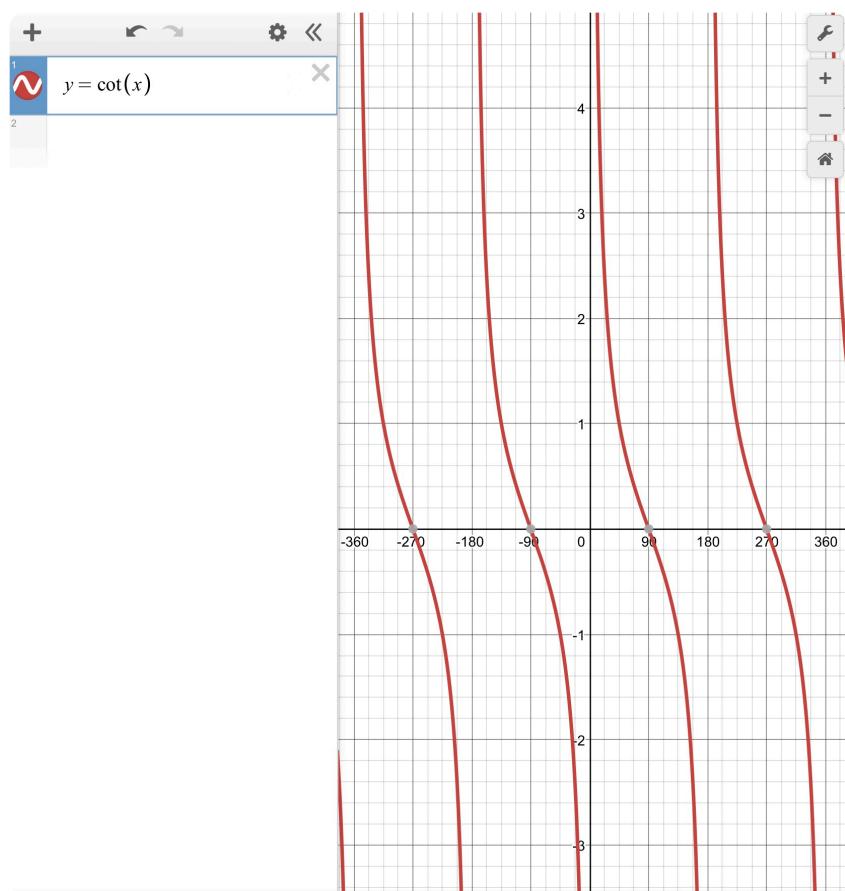
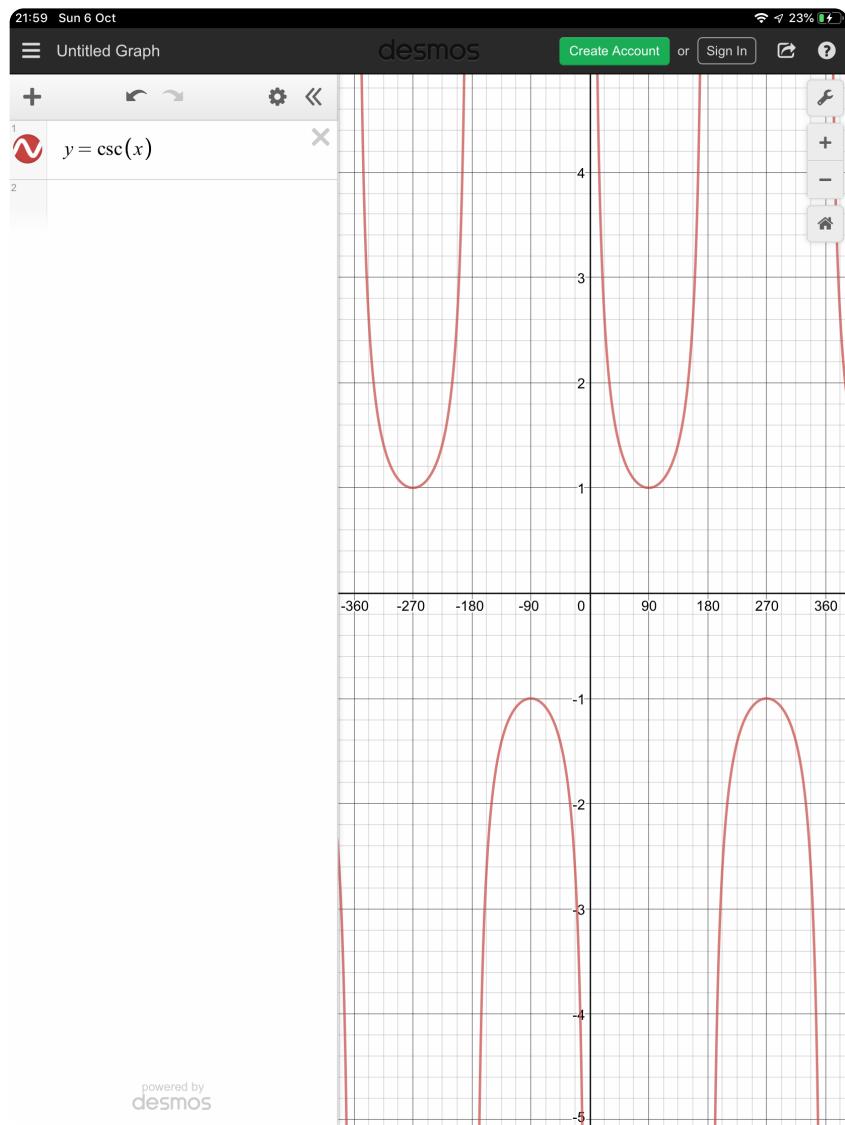
$$1 + x^2 = \frac{dx}{dy}$$

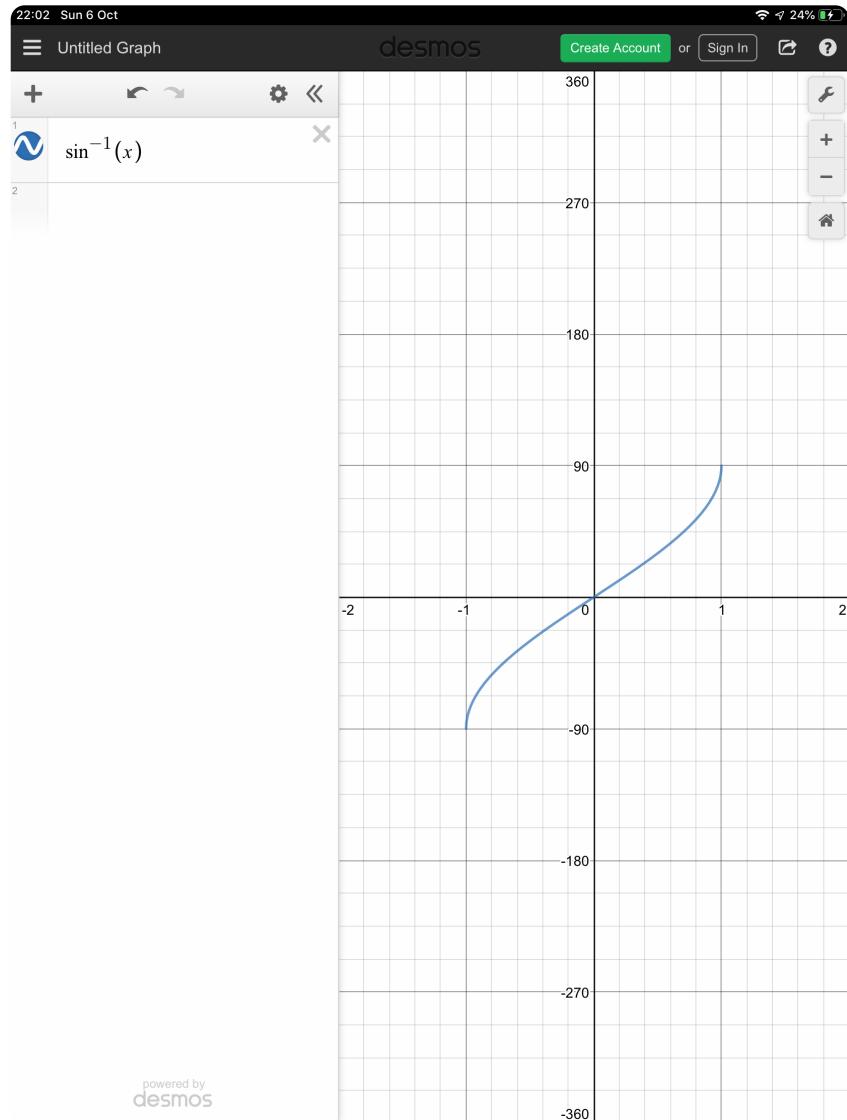
$$\frac{1}{1+x^2} = \frac{dy}{dx}$$

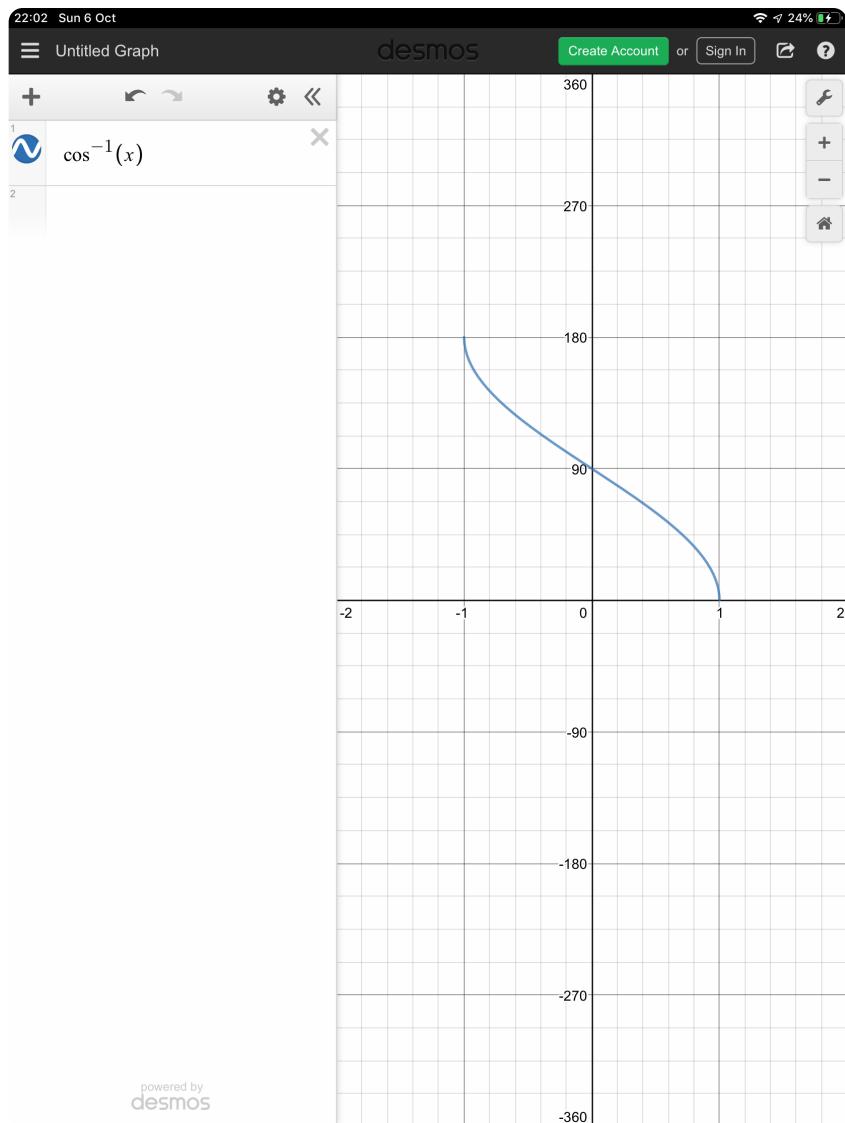
$$\Rightarrow \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

Graphs









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