



- 

The values were

0.68      -0.79      0.08

(6)

A	-0.79	Negative correlation apparent
B	0.08	No correlation apparent
C	0.68	Positive correlation apparent

2. The following table summarises the distances, to the nearest km, that 134 examiners travelled to attend a meeting in London.

Distance (km)	Number of examiners	
41–45	4	$4 \div 5 = 0.8$
46–50	19	$19 \div 5 = 3.8$
51–60	53	$53 \div 10 = 5.3$
61–70	37	$37 \div 10 = 3.7$
71–90	15	$15 \div 20 = 0.75$
91–150	6	$6 \div 60 = 0.1$

- (a) Give a reason to justify the use of a histogram to represent these data.

*Grouped frequency table for continuous data* (1)

- (b) Calculate the frequency densities needed to draw a histogram for these data.

**(DO NOT DRAW THE HISTOGRAM)** (2)

- (c) Use interpolation to estimate the median  $Q_2$ , the lower quartile  $Q_1$ , and the upper quartile  $Q_3$  of these data.

(4)

The mid-point of each class is represented by  $x$  and the corresponding frequency by  $f$ . Calculations then give the following values

$$\Sigma fx = 8379.5 \quad \text{and} \quad \Sigma fx^2 = 557489.75$$

- (d) Calculate an estimate of the mean and an estimate of the standard deviation for these data.

(4)

One coefficient of skewness is given by

$$\frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}$$

- (e) Evaluate this coefficient and comment on the skewness of these data.

(4)

- (f) Give another justification of your comment in part (e).

(1)

*c)*

$134 \div 4 = 33.5$	$Q_1$ at 33.5 item
$134 \div 2 = 67$	$Q_2$ at 67 item
$134 \div 4 \times 3 =$	$Q_3$ at 100.5 item

$$Q_1 = 50.5 + \frac{10.5}{53} \times 10 = 52.48$$



Question 2 continued

$$Q_2 = 50.5 + \frac{44}{53} \times 10 = 58.80$$

$$Q_3 = 60.5 + \frac{24.5}{37} \times 10 = 67.12$$

$$d) \quad \sum fx = 8379.5 \quad \sum fx^2 = 557489.75$$

$$\text{Estimate } \bar{x} = \frac{\sum fx}{\sum f} = \frac{8379.5}{134} = 62.53$$

$$\text{Estimate s.d.} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

$$= \sqrt{\frac{557489.75}{134} - 62.53^2}$$

$$= 15.81$$

$$e) \quad \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1} = \frac{67.12 - 2 \times 58.80 + 52.48}{67.12 - 52.48}$$

$$= 0.1366$$

Distribution has slight positive skew

$$f) \quad \text{mean } 62.53 > \text{median } 58.80$$

indicates positive skew



3. A long distance lorry driver recorded the distance travelled,  $m$  miles, and the amount of fuel used,  $f$  litres, each day. Summarised below are data from the driver's records for a random sample of 8 days.

The data are coded such that  $x = m - 250$  and  $y = f - 100$ .

$$\Sigma x = 130 \quad \Sigma y = 48 \quad \Sigma xy = 8880 \quad S_{xx} = 20\,487.5$$

- (a) Find the equation of the regression line of  $y$  on  $x$  in the form  $y = a + bx$ . (6)
- (b) Hence find the equation of the regression line of  $f$  on  $m$ . (3)
- (c) Predict the amount of fuel used on a journey of 235 miles. (1)

**No longer on syllabus**

**However, still worth doing using these formulae from the old syllabus formulae book**

### Correlation and regression

For a set of  $n$  pairs of values  $(x_i, y_i)$

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The regression coefficient of  $y$  on  $x$  is  $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of  $y$  on  $x$  is  $y = a + bx$  where  $a = \bar{y} - b\bar{x}$

a) 
$$b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{S_{xx}}$$



$$b = \frac{8880 - \frac{130 \times 48}{8}}{20,487.5} = 0.395363$$

$$\bar{x} = \frac{\sum x}{n} = \frac{130}{8} = 16.25$$

$$\bar{y} = \frac{\sum y}{n} = \frac{48}{8} = 6$$

$$a = \bar{y} - b\bar{x} = 6 - 0.3954 \times 16.25 = -0.4253$$

Regression Line  $y = -0.4253 + 0.3954x$

$$y = -0.425 + 0.395x$$


---

b) The data are coded such that  $x = m - 250$  and  $y = f - 100$ .

$$f - 100 = -0.425 + 0.395(m - 250)$$

$$f = -0.425 + 0.395m - 0.395363 \times 250 + 100$$

$$f = 0.734 + 0.395m$$


---

c)  $m = 235$

$$f = 0.734 + 0.395 \times 235$$

$$f = 93.55 \text{ litres}$$

$$f = 93.6 \text{ litres}$$


---

4. Aeroplanes fly from City  $A$  to City  $B$ . Over a long period of time the number of minutes delay in take-off from City  $A$  was recorded. The minimum delay was 5 minutes and the maximum delay was 63 minutes. A quarter of all delays were at most 12 minutes, half were at most 17 minutes and 75% were at most 28 minutes. Only one of the delays was longer than 45 minutes.

An outlier is an observation that falls either  $1.5 \times$  (interquartile range) above the upper quartile or  $1.5 \times$  (interquartile range) below the lower quartile.

- (a) On the graph paper opposite draw a box plot to represent these data. (7)
- (b) Comment on the distribution of delays. Justify your answer. (2)
- (c) Suggest how the distribution might be interpreted by a passenger who frequently flies from City  $A$  to City  $B$ . (1)

a)  $IQR = 28 - 12 = 16 \text{ min}$

Top end outliers above  $28 + \frac{3}{2} \times 16 = 52$

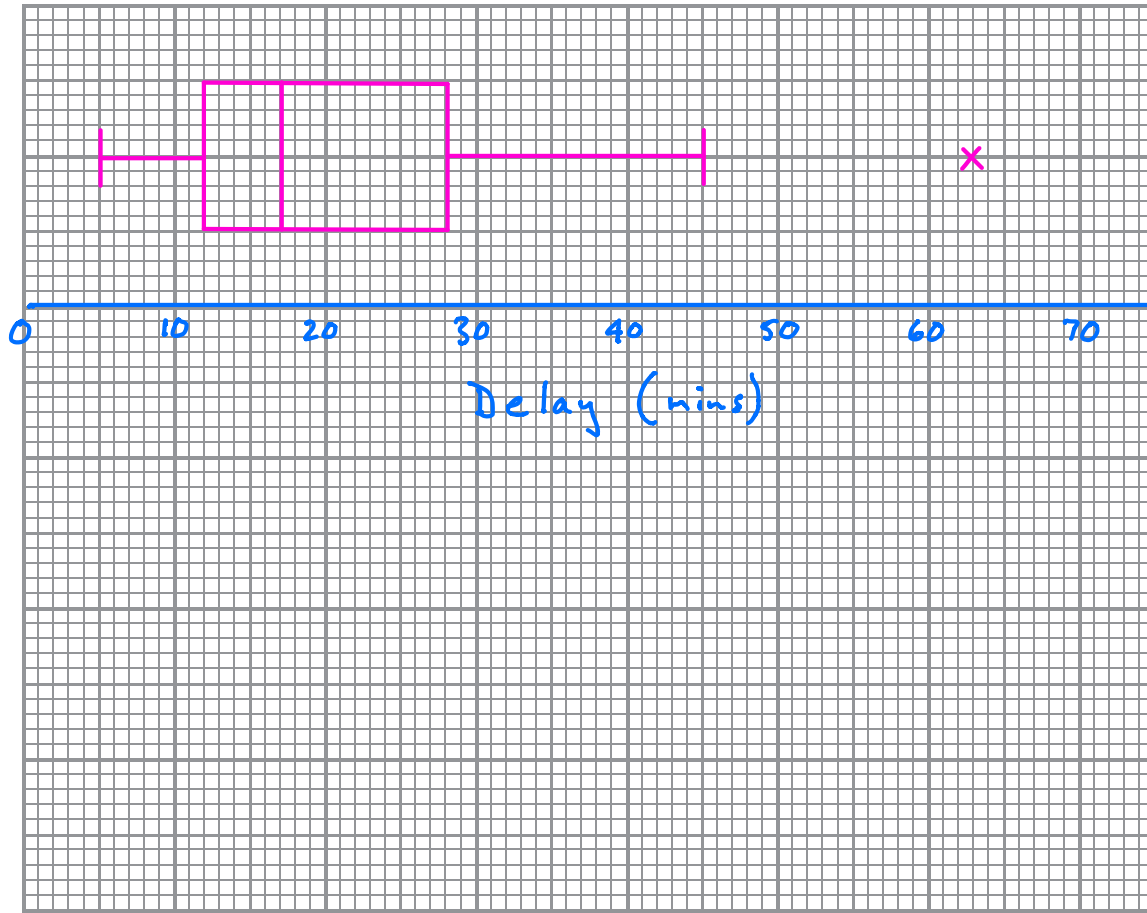
Bottom end outliers below  $12 - \frac{3}{2} \times 16 = -12$

So 1 outlier at top end

0 outliers at bottom end



Question 4 continued



b) Distribution shows positive skew

$$\begin{array}{ccc} Q_3 - Q_2 & > & Q_2 - Q_1 \\ \parallel & & \parallel \\ 11 & > & 5 \end{array}$$

c) Most delays likely to be a reasonably small proportion of total journey time.

Q4

(Total 10 marks)





$$P(X = x) = \begin{cases} kx, & x = 1, 2, 3, \\ k(x+1), & x = 4, 5, \end{cases}$$

No longer on syllabus

- (a) Find the value of  $k$ . (2)
- (b) Find the exact value of  $E(X)$ . (2)
- (c) Show that, to 3 significant figures,  $\text{Var}(X) = 1.47$ . (4)
- (d) Find, to 1 decimal place,  $\text{Var}(4 - 3X)$ . (2)

**Question 5 continued**

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6. A scientist found that the time taken,  $M$  minutes, to carry out an experiment can be modelled by a normal random variable with mean 155 minutes and standard deviation 3.5 minutes.

Find

(a)  $P(M > 160)$ . (3)

(b)  $P(150 \leq M \leq 157)$ . (4)

(c) the value of  $m$ , to 1 decimal place, such that  $P(M \leq m) = 0.30$ . (4)

a) By calc  $P(M > 160) = 0.0766$

b) By calc  $P(150 \leq M \leq 157) = 0.6396$

c) By calc Area beneath  $m = 0.30$

$$m = 153.16$$

$$m = 153.2 \text{ minutes to 1 d.p.}$$



7. In a school there are 148 students in Years 12 and 13 studying Science, Humanities or Arts subjects. Of these students, 89 wear glasses and the others do not. There are 30 Science students of whom 18 wear glasses. The corresponding figures for the Humanities students are 68 and 44 respectively.

A student is chosen at random.

Find the probability that this student

- (a) is studying Arts subjects, (4)

- (b) does not wear glasses, given that the student is studying Arts subjects. (2)

Amongst the Science students, 80% are right-handed. Corresponding percentages for Humanities and Arts students are 75% and 70% respectively.

A student is again chosen at random.

- (c) Find the probability that this student is right-handed. (3)

- (d) Given that this student is right-handed, find the probability that the student is studying Science subjects. (3)

	Sci	Hum	Arts	Total
Glasses	18	44	27	89
No Glasses	12	24	23	59
Total	30	68	50	148

$$a) P(\text{Arts student}) = \frac{50}{148} = \frac{25}{74}$$

$$b) P(\text{No Glasses} \mid \text{Arts}) = \frac{23}{50}$$

$$c) P(\text{Right-handed})$$

$$= \frac{30}{148} \times 0.8 + \frac{68}{148} \times 0.75 + \frac{50}{148} \times 0.7 = \frac{55}{74}$$



## Question 7 continued

$$d) P(\text{Sci} \setminus \text{Right-handed}) = \frac{P(\text{Sci} \cap \text{Right-handed})}{P(\text{Right-handed})}$$

$$= \frac{30}{148} \times 0.8$$

$$\frac{55}{74}$$

$$= \frac{12}{55}$$

Q7

(Total 12 marks)

TOTAL FOR PAPER: 75 MARKS

END

