Upper and Lower Bounds

John runs the 100m in 12.5s. The track is measured to the nearest metre and the time is measured to 1 decimal place

Find the upper and lower bounds for John's speed in m/s.

$$99.5\text{m} \leq \text{Distance} < 100.5\text{m}$$

$$12.45\text{s} \leq \text{Time} < 12.55\text{s}$$

$$\text{Max Speed} = \frac{\text{Dist}}{\text{Time}} = \frac{100.5}{12.45} = 8.07\text{ m/s}$$

$$\text{Min Speed} = \frac{\text{Dist}}{\text{Time}} = \frac{99.5}{12.55} = 7.93\text{ m/s}$$

A carpet measures 4m by 3m with each measurement correct to the nearest 10cm

Find lower and upper bounds for the area of the carpet.

$$3.95\text{m} \leq \text{Length} < 4.05\text{m}$$

$$2.95\text{m} \leq \text{Width} < 3.05\text{m}$$

Upper Bound = $L \times W$ = 4.05×3.05

= $12.35\ m^2$

Lower Bound = $L \times W$ = 3.95×2.95

= $11.65\ m^2$

A plank is 1.8 m long to the nearest 10 cm

60 cm is cut off to the nearest 5 cm.

Find lower and upper bounds for the length of the piece that is left

$1.75m \leq$ Plank $< 1.85m$

$57.5cm \leq$ Cut-off $< 62.5cm$

Lower bound for remainder = $1.75 - 0.625$ (small − large)

= $1.125m$

Upper bound for remainder = $1.85 - 0.575$ (large − small)

= $1.275m$

Exercise

$a = 1.4$ to 1dp

$b = 2.38$ to 2dp

$c = 4.5$ to 1dp

Find bounds for

$$\frac{c - a}{b}$$

$1.35 \leq a < 1.45$

$2.375 \leq b < 2.385$

$4.45 \leq c < 4.55$

Lower bound $= \frac{\text{small}}{\text{large}} = \frac{\text{small } c - \text{large } a}{\text{large } b}$

$= \frac{4.45 - 1.45}{2.385}$

$= 1.25786$ $= 1.26$ to 2 dp

Upper bound $= \frac{\text{large}}{\text{small}} = \frac{\text{large } c - \text{small } a}{\text{small } b}$

$= \frac{4.55 - 1.35}{2.375}$

$= 1.347368$ $= 1.35$ to 2dp