Induction -Matrices

Ex 8C

1) Prove $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 2n \\ 0 & 1 \end{pmatrix}$

$n=1$ $\quad \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2(1) \\ 0 & 1 \end{pmatrix}$ ✓ true for $n=1$

Assume true for $n=k$ then

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}^k = \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix}$$

Consider $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}^{k+1} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}^k \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 2k \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 2+2k \\ 0+0 & 0+1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2(k+1) \\ 0 & 1 \end{pmatrix}$$

Same matrix with k replaced by $k+1$

$\therefore$ if formula true for $n=k$ also true for $n=k+1$

Since true for $n=1$, by mathematical induction it is true for all positive integers n

5) $\underline{M} = \begin{pmatrix} 2 & 5 \\ 0 & 1 \end{pmatrix}$ Prove $\underline{M}^n = \begin{pmatrix} 2^n & 5(2^n - 1) \\ 0 & 1 \end{pmatrix}$

$n = 1$ $\begin{pmatrix} 2 & 5 \\ 0 & 1 \end{pmatrix}^1 = \begin{pmatrix} 2^1 & 5(2^1 - 1) \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 0 & 1 \end{pmatrix}$ ✓

True for $n = 1$

Assume true for $n = k$

then $\underline{M}^k = \begin{pmatrix} 2^k & 5(2^k - 1) \\ 0 & 1 \end{pmatrix}$

Consider $\underline{M}^{k+1} = \underline{M}^k \underline{M}$

$$= \begin{pmatrix} 2^k & 5(2^k - 1) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 5 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2^k \times 2 + 0 & 5 \times 2^k + 5(2^k - 1) \\ 0 + 0 & 0 + 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2^{k+1} & 5(2^k + 2^k - 1) \\ 0 & 1 \end{pmatrix}$$

$2^k + 2^k$

$= 2(2^k)$

$= 2^{k+1}$

$$= \begin{pmatrix} 2^{k+1} & 5(2^{k+1}-1) \\ 0 & 1 \end{pmatrix}$$

Same formula with k replaced by $k+1$

$\therefore$ if true for $n=k$ also true for $n=k+1$ etc.

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b) Find $(\underline{M}^n)^{-1}$

$$\underline{M}^n = \begin{pmatrix} 2^n & 5(2^n-1) \\ 0 & 1 \end{pmatrix}$$

$$(\underline{M}^n)^{-1} = \frac{1}{2^n}\begin{pmatrix} 1 & -5(2^n-1) \\ 0 & 2^n \end{pmatrix}$$

Homework Mixed Exercise 8