DIFFERENTIATION OF POLYNOMIALS

TRANSCRIPT

Basic rule for differentiating a power of x

$\frac{d}{dx} x^n = nx^{n-1}$	$\frac{d}{dx} ax^n = nax^{n-1}$
$\frac{d}{dx} x^6 = 6x^5$	$\frac{d}{dx} 5x^6 = 30x^5$
$\frac{d}{dx} x^3 = 3x^2$	$\frac{d}{dx} 4x^3 = 12x^2$
$\frac{d}{dx} x^2 = 2x$	$\frac{d}{dx} 7x^2 = 14x$
$\frac{d}{dx} x = 1$	$\frac{d}{dx} 3x = 3$
$\frac{d}{dx} c = 0$	$\frac{d}{dx} 8 = 0$

Examples of polynomial differentiation

1. $y = x^4 + 3x^2 \Rightarrow \frac{dy}{dx} = 4x^3 + 6x$

2. $y = 3x^3 - 7x + 2 \Rightarrow \frac{dy}{dx} = 9x^2 - 7$

3. $f(x) = x^2 - 3x - 5 \Rightarrow f'(x) = 2x - 3$

4. $f(x) = x^3 - 2x^2 + 3x \Rightarrow f'(x) = 3x^2 - 4x + 3$

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Note that $\frac{dy}{dx}$ is the derivative of y

and $f'(x)$ is the derivative of $f(x)$

Both types of notation have the same meaning.

Using Autograph we investigated the gradient functions of $y = x^3 - 2x$ and $y = x^4 - 2x^3 - x^2 + 2x$

$y = x^3 - 2x$ $\quad\quad$ $y = x^4 - 2x^3 - x^2 + 2x$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 2$ $\quad\quad$ $\Rightarrow \frac{dy}{dx} = 4x^3 - 6x^2 - 2x + 2$

We found that the gradient function of a cubic function is a quadratic function, and the gradient function of a quartic function is a cubic function.

There now follows an example of a typical exam question.

Example

Consider the curve: $y = x^3 - 2x^2 - x - 12$

i) Find the gradient when $x = 2$

ii) Find the equation of the tangent when $x = 2$

iii) Find the equation of the normal when $x = 2$

i) $$y = x^3 - 2x^2 - x - 12$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x - 1$$

When $x = 2$, $$\frac{dy}{dx} = 3(2)^2 - 4(2) - 1$$
$$= 12 - 8 - 1$$
$$= 3$$

$\therefore$ gradient of curve at $x = 2$ is 3

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ii) When $x = 2$, $y = 2^3 - 2(2)^2 - 2 - 12$
$$= 8 - 8 - 2 - 12$$
$$= -14$$

$\therefore$ point on curve is $(2, -14)$

Tangent has same gradient as curve

Using $y - y_1 = m(x - x_1)$

$$y - -14 = 3(x - 2)$$
$$y + 14 = 3x - 6$$

tangent: $y = 3x - 20$

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iii) Gradient of normal will be $-\frac{1}{3}$ since $3x -\frac{1}{3} = -1$

Using $y - y_1 = m(x - x_1)$

$$y - -14 = -\frac{1}{3}(x - 2)$$

$$y + 14 = -\frac{1}{3}x + \frac{2}{3}$$

$$y = -\frac{1}{3}x + \frac{2}{3} - \frac{42}{3}$$

Normal: $y = -\frac{1}{3}x - \frac{40}{3}$
