Exercise 4A

1 a) $3x^2 + 7x - 4 = 0$

$\alpha + \beta = -\frac{b}{a}$ $\qquad \alpha\beta = \frac{c}{a}$

$\alpha + \beta = -\frac{7}{3}$ $\qquad \alpha\beta = -\frac{4}{3}$

a) $\alpha + \beta = -\frac{7}{3}$

b) $\alpha\beta = -\frac{4}{3}$

c) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-\frac{7}{3}}{-\frac{4}{3}} = \frac{7}{4}$

d) $\alpha^2 + \beta^2$

Aside $(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\Rightarrow \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= \left(-\frac{7}{3}\right)^2 - 2\left(-\frac{4}{3}\right)$

$= +\frac{49}{9} + \frac{8}{3} = \frac{73}{9}$

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3) $6x^2 - 9x + 2 = 0$

$\alpha + \beta = -\frac{-9}{6} = \frac{3}{2}$

$\alpha\beta = \frac{2}{6} = \frac{1}{3}$

a) $\alpha + \beta = \frac{3}{2}$

b) $\alpha^2\beta^2 = (\alpha\beta)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

c) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{3}{2}}{\frac{1}{3}} = \frac{3}{2} \times \frac{3}{1} = \frac{9}{2}$

d) $\alpha^3 + \beta^3$

Aside $(\alpha+\beta)^3 = \alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$

$(\alpha+\beta)^3 = \alpha^3 + \beta^3 + 3\alpha\beta(\alpha+\beta)$

$(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = \alpha^3 + \beta^3$

$\alpha^3 + \beta^3 = \left(\frac{3}{2}\right)^3 - 3\left(\frac{1}{3}\right)\left(\frac{3}{2}\right)$

$= \frac{27}{8} - \frac{3}{2}$

$= \frac{15}{8}$

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5) $ax^2 + bx + c = 0$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\alpha = -\frac{1}{2}$, $\beta = -\frac{1}{3}$ $\quad \alpha+\beta = -\frac{1}{2} + -\frac{1}{3} = -\frac{5}{6}$

$\Rightarrow -\frac{b}{a} = -\frac{5}{6}$

$\Rightarrow \frac{b}{a} = \frac{5}{6}$

$\alpha\beta = \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right) = \frac{1}{6}$

$\Rightarrow \frac{c}{a} = \frac{1}{6}$

$x^2 + \frac{5}{6}x + \frac{1}{6} = 0$

$$6x^2 + 5x + 1 = 0$$

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9) $$nx^2 - (16+n)x + 256 = 0$$

Roots $\alpha, -\alpha$ Find n

$$\alpha + -\alpha = -\frac{-(16+n)}{n} = 0$$

$$\frac{16+n}{n} = 0$$

$$\Rightarrow 16 + n = 0 \qquad \underline{n = -16}$$

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11) $$mx^2 + 4x + 4m = 0$$

roots $k, 2k$

$$\alpha + \beta = 3k = -\frac{4}{m}$$

$$\alpha\beta = 2k^2 = \frac{4m}{m} = 4$$

$$2k^2 = 4$$

$$k^2 = 2$$

$$k = \pm\sqrt{2}$$

If $k = \sqrt{2}$ $\qquad 3\sqrt{2} = -\frac{4}{m}$

$$3\sqrt{2}\,m = -4$$

$$m = -\frac{4}{3\sqrt{2}} = -\frac{2\sqrt{2}}{3}$$

or if $k = -\sqrt{2}$ $\quad -3\sqrt{2} = -\frac{4}{m}$

$$-3\sqrt{2}m = -4$$

$$m = \frac{-4}{-3\sqrt{2}}$$

$$m = \frac{2\sqrt{2}}{3}$$
