7.

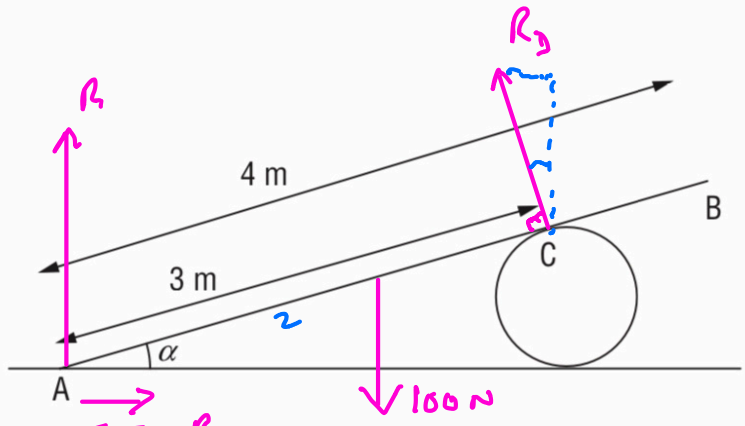
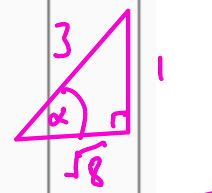



Figure 4




$\cos\alpha = \frac{\sqrt{8}}{3}$

A uniform plank AB, of weight 100 N and length 4 m, rests in equilibrium with the end A on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is C, where AC = 3 m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle $\alpha$ to the horizontal, where $\sin\alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is $\mu$. Modelling the plank as a rod, find the least possible value of $\mu$.

(10)

Resolve ↕

$$100 = R + R_D \cos\alpha$$
$$100 = R + \frac{\sqrt{8}}{3} R_D \qquad (1)$$

Resolve ⟷

$$F = \mu R = R_D \sin\alpha$$
$$\mu R = \frac{1}{3} R_D \qquad (2)$$

Moments about A

$$3R_D = 100 \times 2\cos\alpha$$
$$3R_D = \frac{200\sqrt{8}}{3}$$
$$R_D = \frac{200\sqrt{8}}{9} \qquad (3)$$

Sub for $R_D$ in (1)

$$100 = R + \frac{\sqrt{8}}{3}\left(\frac{200\sqrt{8}}{9}\right)$$

$$100 = R + \frac{1600}{27}$$

$$R = 100 - \frac{1600}{27} = \frac{1100}{27}$$

Sub for $R$ and $R_D$ in (2)

$$\frac{1100}{27}\mu = \frac{1}{3} \times \frac{200\sqrt{8}}{9}$$

$$\frac{1100\mu}{27} = \frac{200\sqrt{8}}{27}$$

$$\mu \geq \frac{200\sqrt{8}}{1100} = \frac{4\sqrt{2}}{11} = 0.514$$

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7.

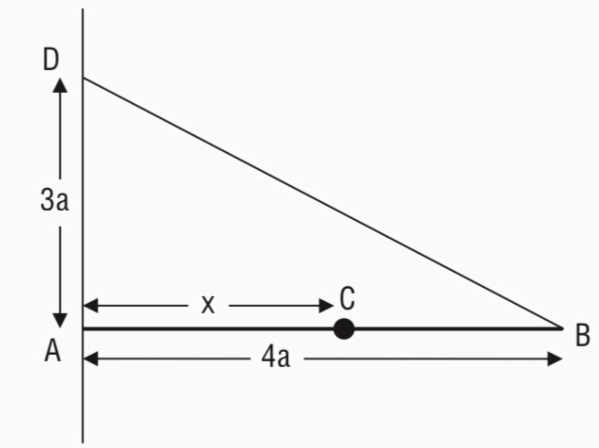


Figure 3

A uniform rod AB, of mass 3m and length 4a, is held in a horizontal position with the end A against a rough vertical wall. One end of a light inextensible string BD is attached to the rod at B and the other end of the string is attached to the wall at the point D vertically above A, where AD = 3a. A particle of mass 3m is attached to the rod at C, where AC = x. The rod is in equilibrium in a vertical plane perpendicular to the wall as shown in Figure 3. The tension in the string is $\frac{25}{4}mg$.

Show that

(a) $x = 3a$,

(5)

(b) the horizontal component of the force exerted by the wall on the rod has magnitude 5mg.

(3)

The coefficient of friction between the wall and the rod is $\mu$. Given that the rod is about to slip,

(c) find the value of $\mu$.

(5)