Variable Acceleration

Displacement

$\downarrow\uparrow$

Velocity

$\downarrow\uparrow$

Acceleration

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt}, \quad a = \frac{d^2s}{dt^2}$$

$$s = \int v\,dt$$

$$v = \int a\,dt$$

SUVAT is consistent with this

$$s = ut + \tfrac{1}{2}at^2$$

$$v = \frac{ds}{dt} = u + at$$

$$a = \frac{dv}{dt} = a$$

$$v = \int a\,dt$$

$$v = u + at$$

(u = velocity when $t = 0$)

$$s = \int v\,dt$$

$$s = ut + \tfrac{1}{2}at^2 + c$$

$$s = ut + \tfrac{1}{2}at^2$$

(if $s = 0$, when $t = 0$)

Exercise 11A

1) $s = 9t - t^3$

a) when $t = 1$, $s = 9(1) - 1^3 = 8$ m

b) when $s = 0$

$$9t - t^3 = 0$$

$$t(9 - t^2) = 0$$

$$t(3 + t)(3 - t) = 0$$

$\Rightarrow t = 0, \quad t = 3$ $\qquad \cancel{t = -3}$

3) $v = 3 + 5t - t^2$

a) $t = 1, \quad v = 3 + 5(1) - 1^2 = 7 \text{ ms}^{-1}$

b) $v = -(t^2 - 5t - 3)$

$v = -\left[\left(t - \frac{5}{2}\right)^2 - 3 - \frac{25}{4} \right]$

$v = -\left[\left(t - \frac{5}{2}\right)^2 - \frac{37}{4} \right]$

$v = -\left(t - \frac{5}{2}\right)^2 + \frac{37}{4} \quad \text{ms}^{-1}$

V_{max} when $t = \frac{5}{2}$, $\quad V_{max} = \frac{37}{4} \text{ ms}^{-1}$

or 9.25 ms^{-1}

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c) $t = 7, \quad v = 3 + 5(7) - 7^2$

$v = 3 + 35 - 49$

$v = -11 \text{ ms}^{-1}$

Direction in negative direction

Exercise 11B

1) a) $s = 4t^4 - \frac{1}{t} = 4t^4 - t^{-1}$

i) $v = 16t^3 + t^{-2} = 16t^3 + \frac{1}{t^2}$

ii) $a = 48t^2 - 2t^{-3} = 48t^2 - \frac{2}{t^3}$

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d) $x = \frac{3t^4 - 2t^3 + 5}{2t}$

$x = \frac{3}{2}t^3 - t^2 + \frac{5}{2}t^{-1}$

i) $v = \frac{9}{2}t^2 - 2t - \frac{5}{2}t^{-2}$

ii) $a = 9t - 2 + 5t^{-3}$

Exercise 11C

1) $s = 0.4t^3 - 0.3t^2 - 1.8t + 5 \qquad 0 \le t \le 3$

a) $v = 1.2t^2 - 0.6t - 1.8$

$a = \frac{dv}{dt} = 2.4t - 0.6$

$V_{max or min}$ when $a = 0$ $\qquad 2.4t - 0.6 = 0$

$2.4t = 0.6$

$t = \frac{0.6}{2.4} = 0.25\,s$

$\frac{d^2v}{dt^2} = +2.4 \qquad \therefore V_{min}$ at $t = 0.25\,s$

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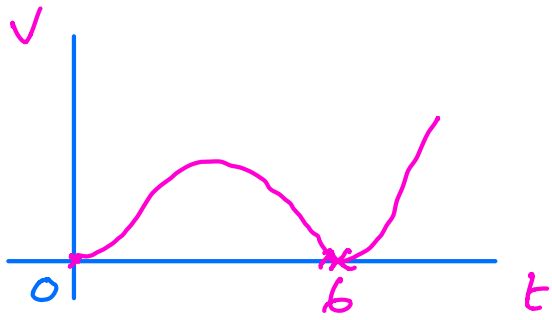
b) $t = 0.25 \quad s = 0.4 \times 0.25^3 - 0.3 \times 0.25^2 - 1.8 \times 0.25 + 5$

$s = 4.54\,m$

$v = 1.2 \times 0.25^2 - 0.6 \times 0.25 - 1.8 = -1.875\,ms^{-1}$

3) $v = t^2(6-t)^2$ $\qquad t \geq 0$



$$v = t^2(36 - 12t + t^2)$$

$$v = 36t^2 - 12t^3 + t^4$$

$$\frac{dv}{dt} = 72t - 36t^2 + 4t^3$$

$$\frac{dv}{dt} = 4t(18 - 9t + t^2)$$

$$\frac{dv}{dt} = 4t(t-6)(t-3)$$

$$t = 0, \quad t = 3, \quad t = 6$$

From graph Max v when $t = 3$

$$v = 3^2(6-3)^2 = 9 \times 9 = 81 \text{ ms}^{-1}$$

Homework Q2, Q4 from Exercises 11A, 11B, 11C