

Revision 3

$$\sqrt{2} \approx 1.414$$

Surds

$$\sqrt{3} \approx 1.732$$

Rules for Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b}$$

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

BUT

$$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$$

$$\sqrt{a} - \sqrt{b} \neq \sqrt{a-b}$$

Task 1

Simplifying

Ex 1

$$\sqrt{27} + \sqrt{12}$$

$$= \sqrt{9 \times 3} + \sqrt{4 \times 3}$$

$$= 3\sqrt{3} + 2\sqrt{3} = 5\sqrt{3}$$

Ex 2

$$\sqrt{2} + \sqrt{50}$$

$$= \sqrt{2} + \sqrt{25 \times 2}$$

$$= \sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$$

Ex 3

$$\sqrt{48} + \sqrt{12} - \sqrt{27}$$

Perfect Squares
 1 4 9 16 25
 36 49 64 81 100

$$= \sqrt{16 \times 3} + \sqrt{4 \times 3} - \sqrt{9 \times 3}$$

$$= 4\sqrt{3} + 2\sqrt{3} - 3\sqrt{3} = 3\sqrt{3}$$

Exercise

Simplify

$$1) \quad \sqrt{20} + \sqrt{45}$$

$$= \sqrt{4 \times 5} + \sqrt{9 \times 5} = 2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$$

$$2) \quad \sqrt{28} + \sqrt{700}$$

$$= \sqrt{4 \times 7} + \sqrt{100 \times 7} = 2\sqrt{7} + 10\sqrt{7} = 12\sqrt{7}$$

$$3) \quad \sqrt{18} + \sqrt{72}$$

$$= \sqrt{9 \times 2} + \sqrt{36 \times 2} = 3\sqrt{2} + 6\sqrt{2} = 9\sqrt{2}$$

$$4) \quad \sqrt{11} + \sqrt{44} + \sqrt{99}$$

$$= \sqrt{11} + \sqrt{4 \times 11} + \sqrt{9 \times 11} = \sqrt{11} + 2\sqrt{11} + 3\sqrt{11} = 6\sqrt{11}$$

$$5) \quad \sqrt{24} + \sqrt{150} - \sqrt{54}$$

$$= \sqrt{4 \times 6} + \sqrt{25 \times 6} - \sqrt{9 \times 6} = 2\sqrt{6} + 5\sqrt{6} - 3\sqrt{6} = 4\sqrt{6}$$

TASK 2 Expanding

$$\text{Ex 1} \quad (3 + \sqrt{2})(5 + \sqrt{2})$$

$$= 15 + 5\sqrt{2} + 3\sqrt{2} + 2$$

$$= 17 + 8\sqrt{2}$$

$$\begin{aligned}\text{Ex2} \quad & (5 - 2\sqrt{3})(4 + 7\sqrt{3}) \\ &= 20 - 8\sqrt{3} + 35\sqrt{3} - 42 \\ &= -22 + 27\sqrt{3}\end{aligned}$$

$$\begin{aligned}\text{Ex3} \quad & (4 + 2\sqrt{3})(3 + 4\sqrt{5}) \\ &= 12 + 6\sqrt{3} + 16\sqrt{5} + 8\sqrt{15}\end{aligned}$$

Exercise

$$\begin{aligned}1) \quad & (7 + \sqrt{5})(2 + \sqrt{5}) \\ &= 14 + 2\sqrt{5} + 7\sqrt{5} + 5 \\ &= 19 + 9\sqrt{5}\end{aligned}$$

$$\begin{aligned}2) \quad & (3 - 2\sqrt{3})(4 + 3\sqrt{3}) \\ &= 12 - 8\sqrt{3} + 9\sqrt{3} - 18 \\ &= -6 + \sqrt{3}\end{aligned}$$

Task 3 Rationalising Denominators

$$\text{Ex1} \quad \frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$$

$$\text{Ex2} \quad \frac{12}{\sqrt{3}} = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\begin{aligned}
 \text{Ex 3)} \quad \frac{3}{5+\sqrt{2}} &= \frac{3}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}} \\
 &= \frac{3(5-\sqrt{2})}{5^2 - \sqrt{2}^2} = \frac{15-3\sqrt{2}}{25-2} \\
 &= \frac{15-3\sqrt{2}}{23} \\
 &\text{or } \frac{15}{23} - \frac{3\sqrt{2}}{23}
 \end{aligned}$$

Exercise

$$1) \quad \frac{20}{\sqrt{5}} = \frac{20}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$$

$$\begin{aligned}
 2) \quad \frac{3+\sqrt{2}}{5-\sqrt{2}} &= \frac{3+\sqrt{2}}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} \\
 &= \frac{15+5\sqrt{2}+3\sqrt{2}+2}{5^2 - \sqrt{2}^2} \\
 &= \frac{17+8\sqrt{2}}{23}
 \end{aligned}$$

Completing The Square

$$\begin{aligned}
 \text{Ex1} \quad & x^2 + 10x + 4 \\
 &= (x+5)^2 + 4 - 25 \\
 &= (x+5)^2 - 21
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex2} \quad & x^2 + 7x + 1 \\
 &= (x + \frac{7}{2})^2 + 1 - \frac{49}{4} \\
 &= (x + \frac{7}{2})^2 - \frac{45}{4}
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex3} \quad & x^2 - 10x + 6 \\
 &= (x - 5)^2 + 6 - 25 \\
 &= (x - 5)^2 - 19
 \end{aligned}$$

Line of symmetry
 $\hookrightarrow y = x^2 - 10x + 6$
 is $x = 5$
 Min point
 $(5, -19)$

Exercise Complete the square

$$\begin{aligned}
 1) \quad & x^2 - 8x + 10 \\
 &= (x - 4)^2 + 10 - 16 = (x - 4)^2 - 6
 \end{aligned}$$

$$\begin{aligned}
 2) \quad & x^2 + 3x + 5 \\
 &= (x + \frac{3}{2})^2 + 5 - \frac{9}{4} = (x + \frac{3}{2})^2 + \frac{11}{4}
 \end{aligned}$$
