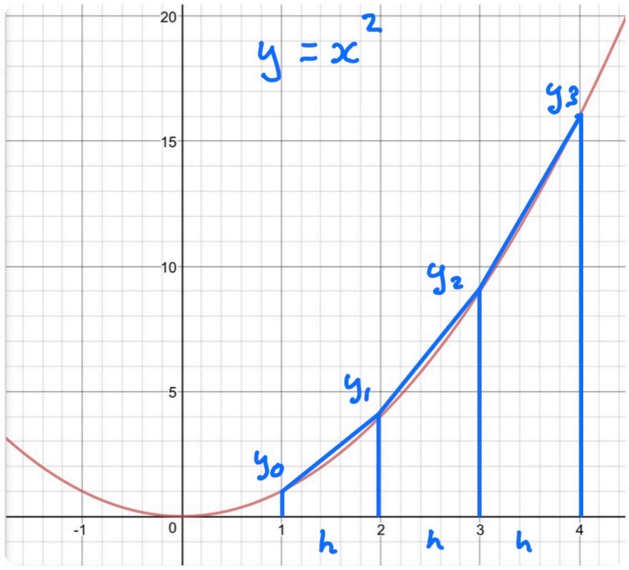
Trapezium Rule

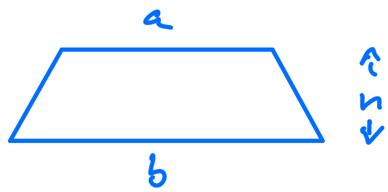
for approximating area under a curve



Approximate the area under the curve between $x = 1$ and $x = 4$

We can do this with the sum of 3 trapezia

Area of trapezium $= \frac{1}{2}(a+b)h$



$$\text{Area} \approx \frac{1}{2}(y_0+y_1)h + \frac{1}{2}(y_1+y_2)h + \frac{1}{2}(y_2+y_3)h$$

$$= \frac{h}{2}\left[y_0+y_1+y_1+y_2+y_2+y_3\right]$$

$$= \frac{h}{2}\left[y_0+2(y_1+y_2)+y_3\right]$$

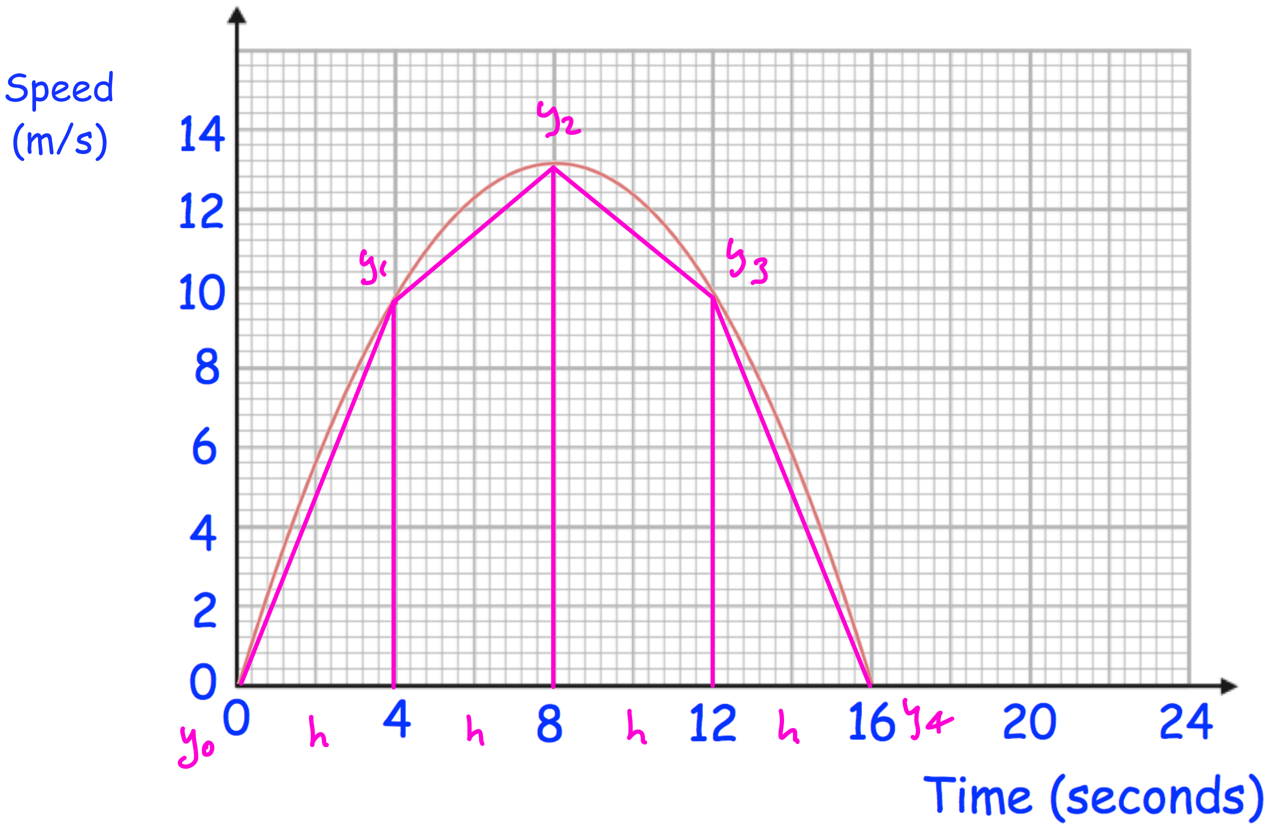
$$= \frac{1}{2}\left[1+2(4+9)+16\right]$$

$$= \frac{1}{2}\left[43\right] = \frac{43}{2} = 21.5$$

In general for n strips

$$\text{Area} \approx \frac{h}{2}\left[y_0+2(y_1+y_2+\dots+y_{n-1})+y_n\right]$$

1. Here is a speed-time graph for a toy rocket.



(a) Work out an estimate for the distance the rocket travelled in the 16 seconds.
Use 4 strips of equal width.

$$\text{Area} \approx \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3) + y_4\right]$$

$$= \frac{4}{2}\left[0 + 2(9.8 + 13.2 + 9.8) + 0\right]$$

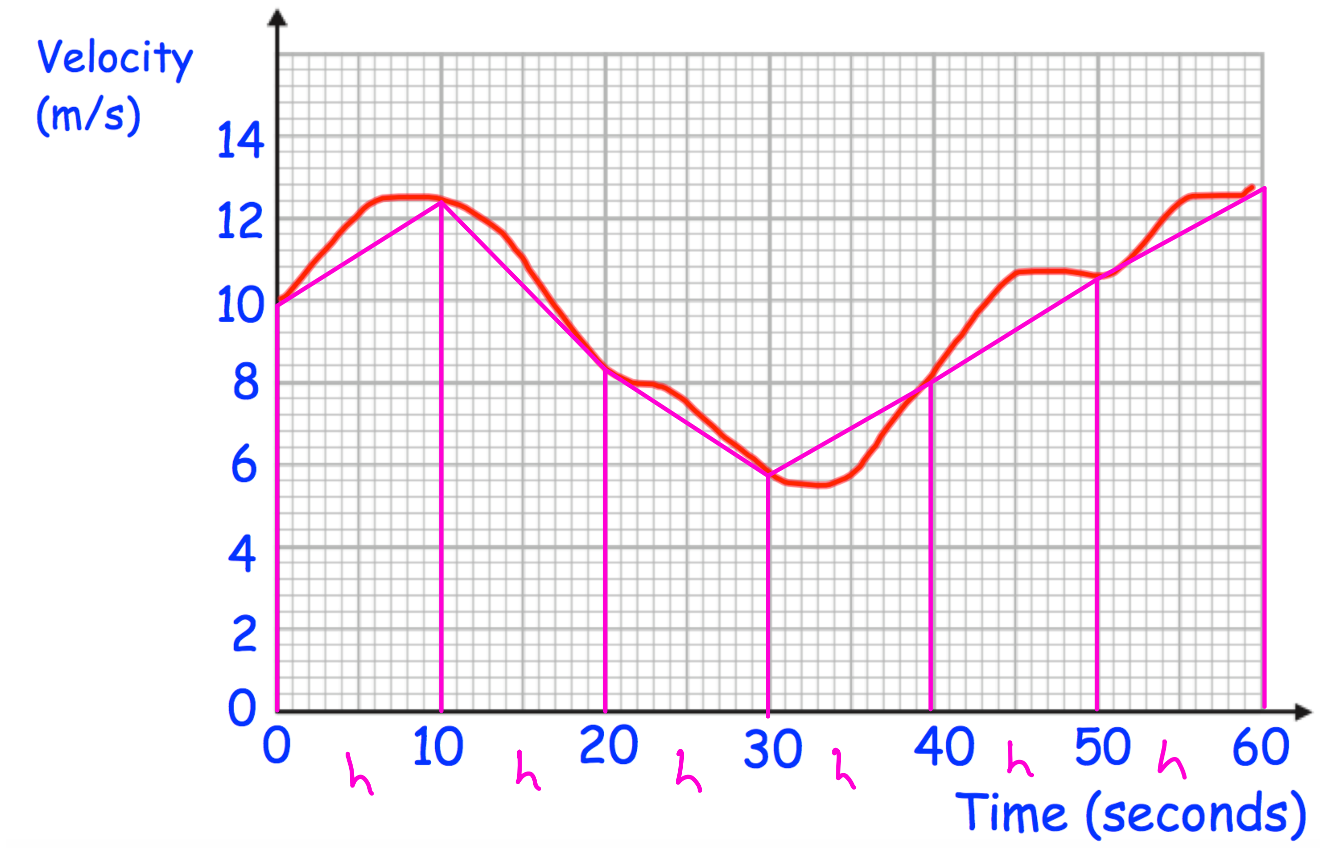
$$= 131.2 \text{ m}$$

......................m
(3)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance the rocket travelled?
Give a reason for your answer

Underestimate because the trapezia are
all below the curve
(1)

2. Here is a velocity time graph for the first 60 seconds of a journey.



Calculate an estimate for the total distance travelled in the 60 seconds.

$$\text{Distance} \approx \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5) + y_6\right]$$

$$= \frac{10}{2}\left[10 + 2(12.4 + 8.4 + 5.6 + 8 + 10.6) + 12.8\right]$$

$$= 564\text{ m}$$

…………………m
(5)