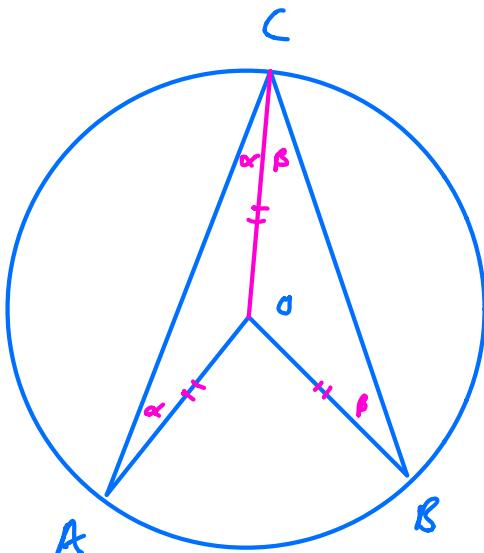


Circle Theorems

Prove $\angle AOB = 2 \times \angle ACB$



$\triangle AOC$ and $\triangle BOC$ are isosceles

Let $\angle OAC = \alpha$

then $\angle ACO = \alpha$ (base angles)

$\angle AOC = 180 - 2\alpha$ (\angle sum of \triangle)

Let $\angle OBC = \beta$

then $\angle OCB = \beta$ (base angles)

then $\angle BOC = 180 - 2\beta$ (\angle sum of \triangle)

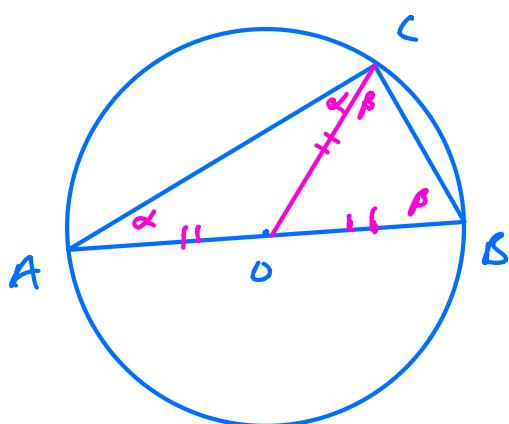
$$\angle AOB + \angle AOC + \angle BOC = 360^\circ \text{ (angles at a point)}$$

$$\angle AOB + 180 - 2\alpha + 180 - 2\beta = 360$$

$$\cancel{\angle AOB} = \cancel{360 - 180 + 2\alpha - 180 + 2\beta}$$

$$\angle AOB = 2\alpha + 2\beta = 2(\alpha + \beta) = 2\angle ACB$$

Prove Angle in Semi-circle = 90°



Prove $\angle ACB = 90^\circ$

$\triangle AOC$ and $\triangle BOC$ are isosceles

Let $\angle OAC = \alpha$

then $\angle OCA = \alpha$ (base angles)

Let $\angle OBC = \beta$

then $\angle OCB = \beta$ (base angles)

Angles of $\triangle ABC$ add to 180°

$$\therefore \alpha + \alpha + \beta + \beta = 180$$

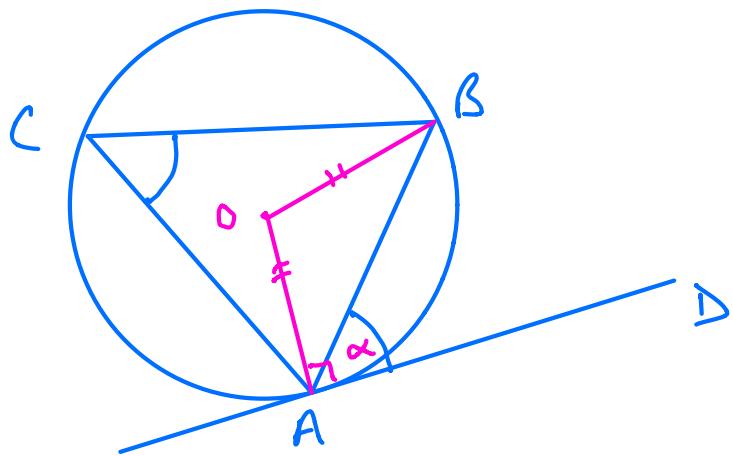
$$2\alpha + 2\beta = 180$$

$$\alpha + \beta = 90^\circ$$

$$\therefore \angle ACB = 90^\circ$$

Prove Alternate Segment Theorem

Prove $\angle DAB = \angle ACB$



Let $\angle BAD = \alpha$

$$\angle BAO = 90 - \alpha \text{ (tangential radius)}$$

$$\angle ABO = 90 - \alpha \text{ (base angles of isosceles } \triangle)$$

$$\begin{aligned}\angle AOB &= 180 - (90 - \alpha) - (90 - \alpha) \\ &= 2\alpha\end{aligned}$$

$$\angle AOB = 2 \times \angle ACB$$

(\angle at centre twice \angle at circ)

$$\therefore \angle ACB = \alpha$$

$$\therefore \angle BAC = \angle ACB$$
