Iterative Sequences (Recurrence Relations)

4. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 2$$
$$a_{n+1} = 3a_n - c$$

where c is a constant.

(a) Find an expression for a_2 in terms of c.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$

(b) find the value of c.

(4)

a)

$$a_2 = 3(2) - c$$
$$\underline{a_2 = 6 - c}$$

b)

$$a_3 = 3(6-c) - c$$
$$\underline{a_3 = 18 - 4c}$$

$$\sum_{i=1}^{3} a_i = 2 + 6 - c + 18 - 4c = 0$$
$$26 - 5c = 0$$
$$26 = 5c$$
$$\frac{26}{5} = c$$
$$\underline{c = 5.2}$$

5. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = k,$$
$$a_{n+1} = 5a_n + 3, \qquad n \geqslant 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k.

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

a) $a_2 = 5k + 3$

b) $a_3 = 5(5k+3) + 3$
$= 25k + 15 + 3$
$= 25k + 18$

c) i) $a_4 = 5(25k+18) + 3$
$= 125k + 90 + 3$
$= 125k + 93$

$\sum_{r=1}^{4} a_r = k + 5k + 3 + 25k + 18 + 125k + 93$
$= 156k + 114$

ii) $156k + 114 = 6(26k + 19)$

6 is a factor of $\sum_{r=1}^{4} a_r$ $\therefore$ it is divisible by 6

4. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1$$

$$x_{n+1} = ax_n + 5, \qquad n \geqslant 1$$

where a is a constant.

(a) Write down an expression for x_2 in terms of a.

(1)

(b) Show that $x_3 = a^2 + 5a + 5$

(2)

Given that $x_3 = 41$

(c) find the possible values of a.

(3)

a) $x_2 = a + 5$

b) $x_3 = a(a+5) + 5$

$x_3 = a^2 + 5a + 5$

c) $a^2 + 5a + 5 = 41$

$a^2 + 5a - 36 = 0$

$(a-4)(a+9) = 0$

$a = 4$ or $a = -9$

5. A sequence of numbers a_1, a_2, a_3 ... is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \qquad (n \geqslant 1)$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2

(1)

(b) Show that $a_3 = 12 - 3c$

(2)

Given that $\sum_{i=1}^{4} a_i \geqslant 23$

(c) find the range of values of c.

(4)

a) $a_2 = 2(3) - c$

$= 6 - c$

b) $a_3 = 2(6 - c) - c$

$= 12 - 2c - c$

$= 12 - 3c$

c) $a_4 = 2(12 - 3c) - c$

$= 24 - 7c$

$\sum_{i=1}^{4} a_i = 3 + 6 - c + 12 - 3c + 24 - 7c$

$= 45 - 11c \geqslant 23$

$45 - 23 \geqslant 11c$

$22 \geqslant 11c$

$\frac{22}{11} \geqslant c$

$c \leq 2$

- **A sequence is increasing if $u_{n+1} > u_n$ for all $n \in \mathbb{N}$.**
- **A sequence is decreasing if $u_{n+1} < u_n$ for all $n \in \mathbb{N}$.**
- **A sequence is periodic if the terms repeat in a cycle. For a periodic sequence there is an integer k such that $u_{n+k} = u_n$ for all $n \in \mathbb{N}$. The value k is called the order of the sequence.**

- 2, 3, 4, 5… is an increasing sequence.
- −3, −6, −12, −24… is a decreasing sequence.
- −2, 1, −2, 1, −2, 1 is a periodic sequence with a period of 2.
- 1, −2, 3, −4, 5, −6… is not increasing, decreasing or periodic.