

Iteration

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3-x_n)}{(3+x_n)}\right)}, n \geq 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1 , x_2 and x_3 .

(3)

$$\begin{aligned} x_1 &= \sqrt{\left(\frac{4(3-1)}{(3+1)}\right)} = \sqrt{\frac{8}{4}} = 1.414 \\ &= 1.41 \quad \text{to } 2dp \end{aligned}$$

$$\begin{aligned} x_2 &= \sqrt{\left(\frac{4(3-1.414)}{(3+1.414)}\right)} = 1.199 \\ &= 1.20 \quad \text{to } 2dp \end{aligned}$$

$$\begin{aligned} x_3 &= \sqrt{\left(\frac{4(3-1.199)}{(3+1.199)}\right)} = 1.30996 \\ &= 1.31 \quad \text{to } 2dp \end{aligned}$$

$$\begin{aligned} x_4 &= \sqrt{\left(\frac{4(3-1.310)}{(3+1.310)}\right)} = 1.252 \\ &= 1.25 \end{aligned}$$

$$f(x) = 3x^3 - 2x - 6$$

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0. \quad (3)$$

(c) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of x_1 , x_2 and x_3 , giving your answers to 4 decimal places.

(3)

$$x_1 = \sqrt{\left(\frac{2}{1.43} + \frac{2}{3}\right)} = 1.437104 \\ = 1.4371 \text{ to 4 dp}$$

$$x_2 = \sqrt{\left(\frac{2}{1.4371} + \frac{2}{3}\right)} = 1.4346966 \\ = 1.4347 \text{ to 4 dp}$$

$$x_3 = \sqrt{\left(\frac{2}{1.4347} + \frac{2}{3}\right)} = 1.43551 \\ = 1.4355$$