Exercises 5C and 5D

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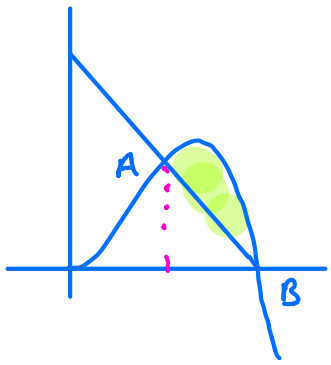

$y = -\frac{1}{2}x^2(x-4)$ (1)

$2x + y = 8$ (2)

In (1)

when $x = 2$, $y = -\frac{1}{2}(2^2)(2-4)$

$= -\frac{1}{2} \times 4 \times -2$

$= 4$

In (2)

when $x = 2$ $\quad 2(2) + y = 8$

$y = 8 - 4$

$y = 4$

$\therefore A(2,4)$ on both curves

$B(4,0)$

$$\text{Vol} = \pi \int_2^4 \left(-\frac{1}{2}x^2(x-4)\right)^2 dx - \text{vol of cone}$$

$$= \pi \int_2^4 \frac{1}{4}x^4(x-4)^2 dx - \frac{1}{3}\pi 4^2 \times 2$$

$$= \pi \int_2^4 \frac{1}{4}x^4(x^2 - 8x + 16)dx - \frac{32\pi}{3}$$

$$= \frac{\pi}{4}\int_2^4 \left(x^6 - 8x^5 + 16x^4\right)dx - \frac{32\pi}{3}$$

$$= \frac{\pi}{4}\left[\frac{x^7}{7} - \frac{8x^6}{6} + \frac{16x^5}{5}\right]_2^4 - \frac{32\pi}{3}$$

$$= \frac{\pi}{4}\left[\left(\frac{4^7}{7} - \frac{8(4)^6}{6} + \frac{16(4)^5}{5}\right) - \left(\frac{2^7}{7} - \frac{8(2)^6}{6} + \frac{16(2)^5}{5}\right)\right] - \frac{32\pi}{3}$$

$$= \frac{2048}{105}\pi$$

Exercise 5D

Q5 $y = 0.02x^3$

a) $18 = 0.02x^3$

$$x^3 = \frac{18}{0.02}$$

$$x = 9.655$$

Diameter $= 19.3$ cm

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b) $\text{Vol} = \pi\int_0^{18} x^2\,dy$

$$= \pi\int_0^{18} (50y)^{2/3}\,dy$$

$$= 50^{2/3}\pi\int_0^{18} y^{2/3}\,dy$$

$$= 50^{2/3}\pi \left[\frac{3}{5}y^{5/3}\right]_0^{18}$$

$$= 50^{2/3}\pi \times \frac{3}{5} \times 18^{5/3}$$

$$= 3162.8\ cm^3$$

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c) $$\text{Area} \int_0^{12} x\,dy = \int_0^{12} (50y)^{\frac{1}{3}}\,dy$$

$$= 50^{\frac{1}{3}} \left[\frac{3}{4}y^{4/3}\right]_0^{12}$$

$$= 50^{\frac{1}{3}} \times \frac{3}{4} \times 12^{4/3}$$

$$= 75.9\ cm^2$$

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d) Vol of paddle rotation

$$= 50^{2/3}\pi \left[\frac{3}{5}y^{5/3}\right]_0^{12}$$

$$= 1609.1\ cm^3$$

$$\frac{1609.1}{3162.8} = 50.9\ \%$$
