1 A mother, Maisie, and her young daughter, Dora, are both enjoying a fairground ride.

They are each sat on a different model horse, and are each travelling in horizontal circles around the same centre point, with a maximum angular velocity, ω rad s^{-1}.

Their circular paths do not have the same radius.

For safety reasons, based on the motion of the ride, Maisie was asked to sit on a horse that was further from the centre point.

a Using a suitable mathematical model, state with justification why this safety advice was wise, with reference to

i the speed of the model horses,

ii the forces acting on the model horses.

You should list two key assumptions you have made in your reasoning. **(4 marks)**

Maisie takes the advice and sits on the model horse further from the centre point. Her model horse travels in a circle with radius 1 metre greater than the radius of the circle described by Dora's model horse. Maisie is four times heavier than Dora. At maximum angular velocity, Maisie experiences a force towards the centre of rotation that is six times greater than that which Dora experiences.

b Find the distance of Maisie's model horse from the centre point. **(4 marks)**

2 A town planner has created a new horizontal road, which includes a curved corner modelled by a quarter circle of radius r.
She knows that the coefficient of friction between any vehicle and the new road will be 0.75 in wet conditions.

a Show that the maximum safe speed, v_{max}, for any vehicle to travel around this curved corner, without slipping, is given by $v_{max} = \frac{\sqrt{3}}{2}(gr)^{\frac{1}{2}}$, where g is the constant of acceleration due to gravity. **(4 marks)**

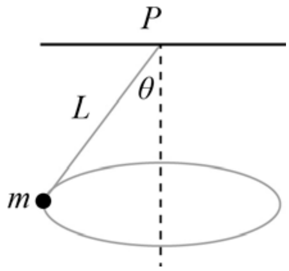
The planner ensures that a speed limit sign, equal to 10% below v_{max}, is visible.
A driver sees the speed limit sign of 72 km/h and takes the curve at this constant speed.

b Find the value of v_{max} in ms^{-1} **(2 marks)**

c Find the time, in seconds to 3 significant figures, for the driver to travel along this curved corner. **(4 marks)**

3 A ball of mass m kg is attached to one end of a light, inextensible string of length L metres. The other end of the string is attached to a fixed point, P. The ball moves with constant speed, v ms^{-1}, in a horizontal circle. The string remains taut and the angle between the string and the vertical is θ. The centre of the circle lies vertically below P. This is shown in Figure 1.

Figure 1



a Show that the period of revolution of the ball is $2\pi\sqrt{\frac{L\cos\theta}{g}}$ seconds. **(6 marks)**

b If this period of revolution is 2 seconds, explain why $L > \frac{g}{\pi^2}$ **(4 marks)**

c Explain how one assumption in this model could affect the validity of this inequality. **(2 marks)**