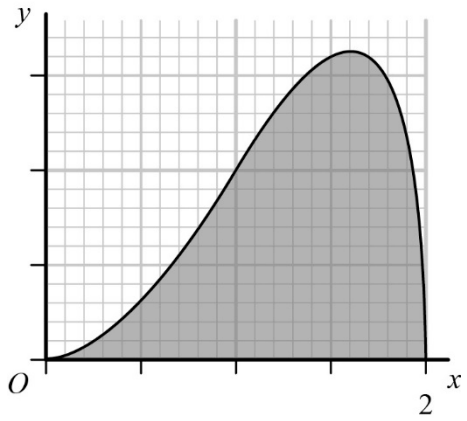
1 Figure 1 shows part of the curve with equation $y = x^2\sqrt{2-x}$. The shaded region is bounded by the curve and the *x*-axis. The region is rotated 2π radians about the *x*-axis to generate a solid of revolution

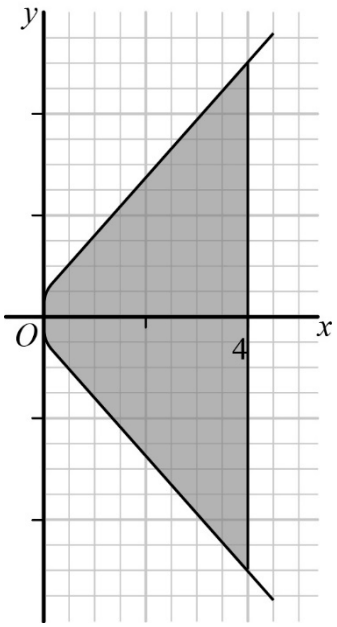
Find the exact volume of solid generated. Leave your answer in the form $a\pi$, where *a* is a rational number to be found. **(4 marks)**

Figure 1



2 Figure 2 shows the curve with equation $2y^2 + x = 3x^2 + \sqrt{5x}$. The shaded region is bounded by the curve and the line *x* = 4. The region is rotated 2π radians about the *x*-axis to generate a solid of revolution. Find the exact volume of the solid generated. Leave your answer in the form $\left(a + b\sqrt{5}\right)\pi$, where *a* and *b* are rational numbers to be found. **(5 marks)**

Figure 2



3 Figure 3 shows a vase with a base width of 8 cm and a height of 12 cm. The edge of the vase is modelled by the equation $y = 3x^2 - 24x + 48$, $2 \leqslant x \leqslant 4$, as shown in Figure 4. The vase is formed by rotating the shape through 360° about the *y*-axis.

Figure 3

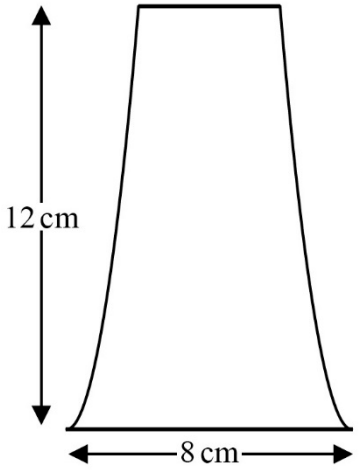
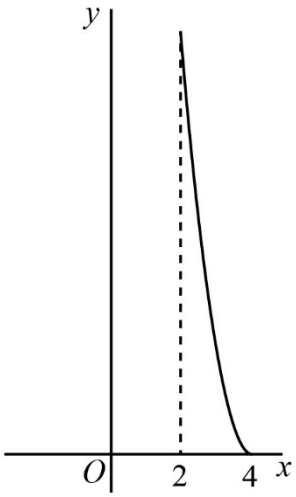


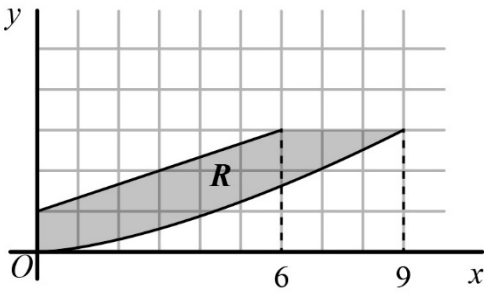
Figure 4



Find the maximum capacity of the vase. Leave your answer in the form $a\pi$, where *a* is a rational number to be found. **(6 marks)**

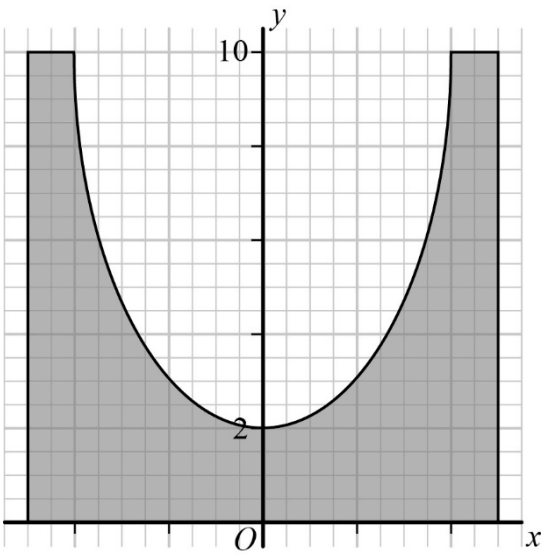
4 Figure 5 shows the shaded region *R* is bounded the curve $y = \frac{1}{9}x^{\frac{3}{2}}$, $0 \leqslant x \leqslant 9$, and the straight line with equation $3y - x - 3 = 0$, $0 \leqslant x \leqslant 6$, the *y*-axis and the line $y = 3$. Find the exact volume of the solid generated when this region is rotated through 2π radians about the *y*-axis. Leave your answer in the form $a\pi$, where *a* is a rational number to be found. **(8 marks)**

Figure 5



5 Figure 6 shows part of an ellipse with equation $\frac{x^2}{16}+\frac{(y-10)^2}{64}=1$ inside a cylinder with a diamater of 10 cm and a height of 10 cm. The shaded region is rotated 360° about the *y*-axis to generate a solid of revolution. Find the exact volume of solid generated. Leave your answer in the form $a\pi$, where *a* is a rational number to be found. **(7 marks)**

Figure 6



6 Figure 7 shows a rubber flotation device. The dimensions of the flotation device are shown in the figure. Figure 8 shows a scale model of the flotation device bounded by curves with equations $x = y^4 + 1$ and $x = -y^4 + 3$. The flotation device is formed by rotating the shaded region through 2π radians about the y-axis.

a Find the volume of the model. **(7 marks)**

b Hence find the volume of the flotation device. **(2 marks)**

It was later discovered that the maximum capacity of air in the flotation device was $32\,000\pi^2$ cm^3.

c Using this information and the correct answer from part **b**, evaluate the model, explaining your reasoning. **(1 mark)**

Figure 7

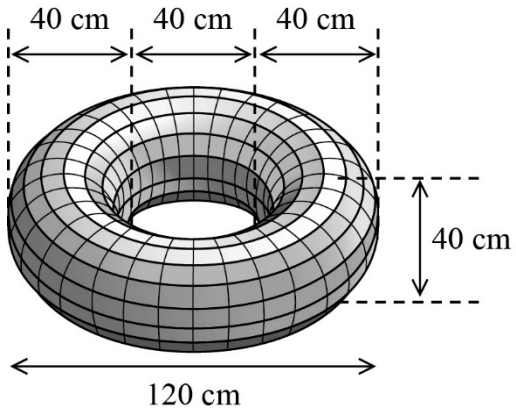
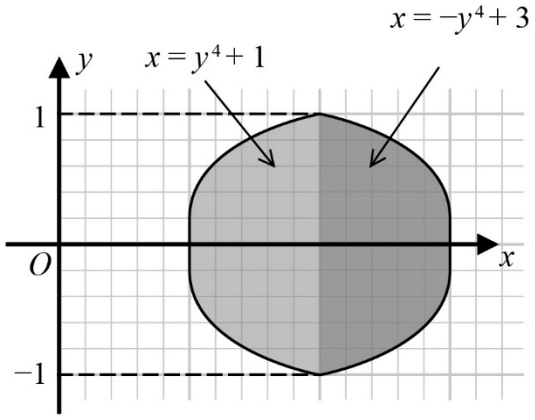


Figure 8



7 A candlestick has base diamater 8 cm and height 28 cm, as shown in Figure 9. A model of the candlestick is shown in Figure 10, together with the equations that were used to create the model.

Figure 9

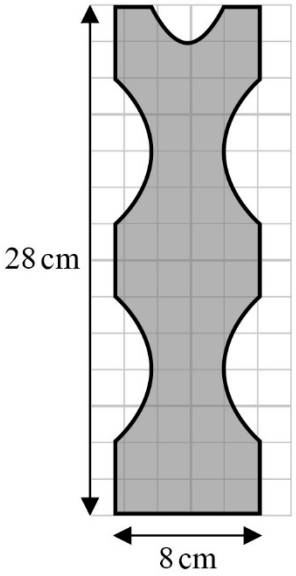
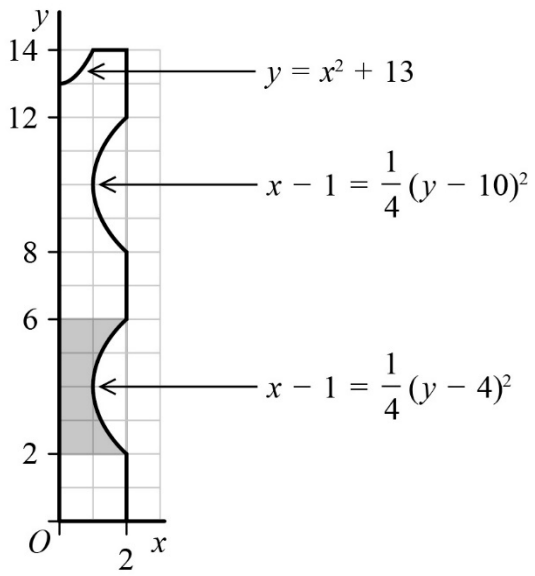


Figure 10



a Show that the volume generated by rotating the shaded region (in Figure 10) 2π radians about the y-axis is $\frac{112}{15}\pi$ **(4 marks)**

b Hence find the volume of metal needed to create the candlestick. **(6 marks)**