Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1a	Let $n=1$ $\sum_{r=1}^{1} r(r+1)=2$ and $\frac{1}{3}n(n+1)(n+2)=\frac{1}{3}(1)(2)(3)=2$ Therefore, the statement is true for $n=1$	**B1**	2.2a	TBC
	Assume general statement is true for $n=k$ So assume, $\sum_{r=1}^{k} r(r+1)=\frac{1}{3}k(k+1)(k+2)$	**M1**	2.4	
	Begins to build an expression for $n = k + 1$ So, $\sum_{r=1}^{k+1} r(r+1)=\sum_{r=1}^{k} r(r+1)+(k+1)(k+2)$ Therefore, $\sum_{r=1}^{k+1} r(r+1)=\frac{1}{3}k(k+1)(k+2)+(k+1)(k+2)$	**M1**	2.1	
	Factorises and arrives at the intended expression. $\frac{1}{3}(k+1)(k+2)[k+3]$	**A1**	1.1b	
	Demonstrates an understanding of the process of mathematical induction Then the general statement is true for $n=k+1$ As the general result has been shown to be true for $n=1$, then the general result is true for all $n \in Z^+$	**A1**	2.4	
		(5)		
1b	Makes an attempt to substitute '$2n$ – 1' into their expression from **1a** $\sum_{r=1}^{2n-1} r(r+1)=\frac{1}{3}(2n-1)(2n-1+1)(2n-1+2)$ oe	**M1**	1.1b	TBC
	Simplifies to obtain $\frac{1}{3}(2n-1)(2n)(2n+1)=\frac{2}{3}n(4n^2-1)$	**A1**	1.1b	
		(2)		
				(7 marks)

Notes

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2	$n=1$, $\sum_{r=1}^{1}\frac{r-1}{r!}=0$ and $\frac{n!-1}{n!}=\frac{1!-1}{1!}=0$	B1	2.2a	TBC
	Assume the general statement is time for $n = k$ So assume $\sum_{r=1}^{k}\frac{r-1}{r!}=\frac{k!-1}{k!}$ is true.	M1	2.4	
	Begins to build an expression for $n = k + 1$ So, $\sum_{r=1}^{k+1}\frac{r-1}{r!}=\sum_{r=1}^{r}\frac{r-1}{r}+\frac{k+1-1}{(k+1)!}$ Therefore $\sum_{r=1}^{k+1}\frac{r-1}{r!}=\frac{k!-1}{k!}+\frac{k+1-1}{(k+1)!}$	M1	2.1	
	Multiplies $\frac{k!-1}{k!}$ by $\frac{k+1}{k+1}$ to obtain a common denominator, $\frac{(k!-1)(k+1)}{k!(k+1)}+\frac{(k+1)!-(k+1)}{(k+1)!}$	M1	1.1b	
	Attempts to simplify: $\frac{(k+1)!-(k+1)}{(k+1)!}+\frac{(k+1)-1}{(k+1)!}=\frac{(k+1)!-1}{(k+1)!}$	M1	1.1b	
	Demonstrates an understanding of the process of mathematical induction, Then the general statement is true for $n=k+1$. As the general result has been shown to be true for $n=1$, then the general result is true for all $n\in Z^{+}$	A1	2.4	
		(6)		
				(6 marks)

Notes

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3	Let $f(n) = 5^{2n} + 11$, where $n \in Z^+$ Therefore, $f(1) = 5^{2n} + 11 = 5^2 + 11 = 36$. 36 is divisible by 6	B1	2.2a	TBC
	Assume statement is true for $n = k$ So, assume $f(k) = 5^{2k} + 11$ is divisible by 6	M1	2.4	
	Begins to build an expression for $n = k + 1$: $f(k+1) = 5^{2(k+1)} + 11$	M1	2.1	
	Use properties of laws of indices in an attempt to simplify, $f(k+1) = 5^{2k+2} + 11 = 5^{2k} * 5^2 + 11 = 25\left(5^{2k}\right) + 11$	M1	1.1b	
	Recognises the need to find $f(k+1) - f(k)$ and simplifies, $f(k+1) - f(k) = 25\left(5^{2k}\right) + 11 - \left(5^{2k} + 11\right) = 24\left(5^{2k}\right)$ $f(k+1) - f(k) = 6 \times \left(4 \times 5^{2k}\right)$	A1	1.1b	
	Therefore,f(n) is divisble by 6 when $n = k + 1$	B1	2.4	
	Demonstrates an understanding of the process of mathematical induction, If f(n) is divisible by 6 when $n = k$, then it has been shown that f(n) is also divisible by 6 when $n = k + 1$ As f(n) is divisible by 6 when n =1, f(n) is also divisible by 6 for all $n \geqslant 1$ and $n \in Z^+$ by mathematical induction	A1	2.4	
		(7)		
				(7 marks)

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4	Let $f(n) = 11^n - 7^n$, where $n \in Z^+$ Therefore, $f(1) = 11^1 - 7^1 = 11 - 7 = 4$ 4 is divisible by 4	**B1**	2.2a	TBC
	Assume statement is true for $n = k$ So,assume $f(k) = 11^k - 7^k$ is divisible by 4	**M1**	2.4	
	Begins to build an expression for $n = k + 1$, $f(k+1) = 11^{k+1} - 7^{k+1} = 11(11^k) - 7(7^k)$	**M1**	2.1	
	Recognises the need to find $f(k+1) - f(k)$ and simplifies, $f(k+1) - f(k) = 11(11^k) - 7(7^k) - (11^k - 7^k)$ $= 10(11^k) - 6(7^k)$	**M1**	1.1b	
	Use expression for $f(k+1) - f(k)$ to find an expression for $f(k+1)$ Therefore, $f(k+1) = f(k) + 10(11^k) - 6(7^k)$ $= f(k) + 6(11^k) - 6(7^k) + 4(11^k)$ $= f(k) + 6f(k) + 4(11^k)$ $= 7f(k) + 4(11^k)$	**A1**	1.1b	
	Therefore, f(n) is divisible by 4 when $n = k + 1$	**B1**	2.4	
	Demonstrates an understanding of the process of mathematical induction: If f(n) is divisible by 4 when $n = k$, then it has been shown that f(n) is also divisible by 4 when $n = k + 1$. As f(n) is divisible by 4 when $n = 1$, f(n) is also divisible by 4 for all $n \geqslant 1$ and $n \in Z^+$ by mathematical induction.	**A1**	2.4	
		(7)		
				(7 marks)

Notes

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5	Let $\mathrm{f}(n) = n^3 + 9n^2 + 5n$, where $n \in Z^+$ Therefore, $\mathrm{f}(1) = (1)^3 + 5(1) + 9(1)^2 = 15$ 15 is divisible by 3	**B1**	2.2a	TBC
	Assume statement is true for $n = k$ So, assume $\mathrm{f}(k) = k^3 + 9k^2 + 5k$ is divisible by 3	**M1**	2.4	
	Begins to build an expression for $n = k + 1$, $\mathrm{f}(k+1) = (k+1)^3 + 9(k+1)^2 + 5(k+1)$ $= k^3 + 3k^2 + 3k + 1 + 9k^2 + 18k + 9 + 5k + 5$ $= k^3 + 12k^2 + 26k + 15$	**M1**	2.1	
	Recognises the need to find $\mathrm{f}\left(k+1\right) - \mathrm{f}\left(k\right)$ and simplifies, $\mathrm{f}(k+1) - \mathrm{f}(k) = \left(k^3 + 12k^2 + 26k + 15\right) - \left(k^3 + 9k^2 + 5k\right)$ $= 3k^2 + 21k + 15$	**M1**	1.1b	
	Use expression for $\mathrm{f}\left(k+1\right) - \mathrm{f}\left(k\right)$ to find an expression for $\mathrm{f}\left(k+1\right)$, $\mathrm{f}(k+1) = \mathrm{f}(k) + 3k^2 + 21k + 15$ $\mathrm{f}(k+1) = \mathrm{f}(k) + 3\left(k^2 + 7k + 2\right)$	**A1**	1.1b	
	Therefore f(n) is divisible by 3 when $n = k + 1$	**B1**	2.4	
	Demonstrates an understanding of the process of mathematical induction, If f(n) is divisible by 3 when $n = k$, then it has been shown that f(n) is also divisible by 3 when $n = k + 1$ As f(n) is divisible by 3 when $n = 1$, f(n) is also divisible by 4 for all $n \geqslant 1$ and $n \in Z^+$ by mathematical induction	**A1**	2.4	
		(7)		
				(7 marks)

Notes

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Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
6	Let $n=1$ LHS = $\begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}^1 = \begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}$ RHS = $\begin{pmatrix}1+1 & -1\\1 & -(1-1)\end{pmatrix} = \begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}$ As LHS = RHS, the matrix equation is true for n = 1.	**B1**	2.2a	TBC
	Assume statement is true for n = k. So assume $\begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}^k = \begin{pmatrix}k+1 & -k\\k & -(k-1)\end{pmatrix}$	**M1**	2.4	
	Begins to build an expression for n = k + 1: $\begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}^{k+1} = \begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}^k \begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}$ $= \begin{pmatrix}k+1 & -k\\k & -(k-1)\end{pmatrix}\begin{pmatrix}2 & -1\\1 & 0\end{pmatrix}$	**M1**	2.1	
	Multiplies the matrices together and simplifies. $\begin{pmatrix}k+1 & -k\\k & -(k-1)\end{pmatrix}\begin{pmatrix}2 & -1\\1 & 0\end{pmatrix} = \begin{pmatrix}2k+2-k & -(k+1)\\2k-(k-1) & -k\end{pmatrix}$ $= \begin{pmatrix}(k+1)+1 & -(k+1)\\k+1 & -((k+1)-1)\end{pmatrix}$	**M1**	1.1b	
	Makes correct conclusion Therefore, the matrix equation is true when $n=k+1$	**B1**	2.4	
	Demonstrates an understanding of the process of mathematical induction, If the matrix equation is true for $n=k$, then it is shown to be true for $n=k+1$ As the matrix equation is true for $n=1$ and $n \in Z^+$ by mathematical induction	**A1**	2.4	
		(6)		
				(6 marks)

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Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	Let $n=1$ LHS $=\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}^1=\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}$ RHS $=\begin{pmatrix}3^1 & 0\\\frac{1}{2}(3^1-1) & 1\end{pmatrix}=\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}$ As LHS = RHS, the matrix equation is true for n = 1.	**B1**	2.2a	TBC
	Assume statement is true for $n = k$. So assume $\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}^k=\begin{pmatrix}3^k & 0\\\frac{1}{2}(3^k-1) & 1\end{pmatrix}$	**M1**	2.4	
	Begins to build an expression for $n = k + 1$: $\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}^{k+1}=\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}^k\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}=\begin{pmatrix}3^k & 0\\\frac{1}{2}(3^k-1) & 1\end{pmatrix}\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}$	**M1**	2.1	
	Correctly multiplies the matrices together, but does not have the correct form for the row 2, column 1 expression. $\begin{pmatrix}3^k & 0\\\frac{1}{2}(3^k-1) & 1\end{pmatrix}\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}=\begin{pmatrix}3\times3^k & 0\\3\times\frac{1}{2}(3^k-1)+1 & 1\end{pmatrix}$	**M1**	1.1b	
	Simplifies the $3\times\frac{1}{2}\left(3^k-1\right)-1$ term so that it is in the correct form, $3\times\frac{1}{2}\left(3^k-1\right)-1=\frac{1}{2}\times3\times3^k-\frac{1}{2}\times3\times1+1=\frac{1}{2}\times3^{k+1}-\frac{1}{2}$ $=\frac{1}{2}\left(3^{k+1}-1\right)$ (continued)	**M1**	1.1b	

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	Sates the correct version of $\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}^{k+1}$, $\begin{pmatrix}3^k & 0\\ \frac{1}{2}(3^k-1) & 1\end{pmatrix}\begin{pmatrix}3 & 0\\1 & 1\end{pmatrix}=\begin{pmatrix}3^{k+1} & 0\\ \frac{1}{2}(3^{k+1}-1) & 1\end{pmatrix}$			
	Therefore the matrix equation is true when $n=k+1$ Demonstrates an understanding of the process of mathematical induction: If the matrix equation is true for $n=k$, then it is shown to be true for $n=k+1$. As the matrix equation is true for $n=1$ and $n\in Z^+$ by mathematical induction.	**B1** **A1**	2.4 2.4	
		(7)		
7b	$\det \mathbf{M} = ad-bc=3^n-0\left(=3^n\right)$ $\left(\mathbf{M}^n\right)^{-1}=\frac{1}{3^n}\begin{pmatrix}1 & 0\\ -\frac{1}{2}(3^n-1) & 3^n\end{pmatrix}$	**A1** **A2**	1.1b 1.1b	TBC
		(3)		
				(10 marks)

Notes

7b: 1 mark award for stating $\frac{1}{\det \mathbf{M}}=\frac{1}{3^n}$ (or dividing each term by 3^n) and 1 mark for switching '*a*' and '*d*' and negating '*b*' and '*c*'.

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