Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1a	Demonstrates understanding of the need to multiply by the complex conjugate; for example, $\frac{5-2\text{i}}{1+3\text{i}}\times\frac{1-3\text{i}}{1-3\text{i}}$ is seen	**M1**	2.5	TBC
	Attempts to multiply these terms; for example, $\frac{5-15\text{i}-2\text{i}+6\text{i}^2}{1-9\text{i}^2}$ is seen	**M1**	1.1b	
	$=-\frac{1}{10}-\frac{17}{10}\text{i}$	**A1**	1.1b	
		(3)		
1b	Calculates $z_2^2=(1+3\text{i})(1+3\text{i})=1+6\text{i}+9\text{i}^2=-8+6\text{i}$	**M1**	1.1b	TBC
	Calculates $\left\|z_2^2\right\|=\sqrt{(-8)^2+6^2}=10$	**A1**	1.1b	
		(2)		
				(5 marks)

Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
2a	Attempts to find the determinant, $\det \mathbf{M} = p(p+1)-(1\times -4)$	**M1**	1.1b	TBC
	Correctly writes $\det \mathbf{M} = p^2+p+4$	**A1**	1.1b	
		(2)		
2b	Completes the square, $\det \mathbf{M} = \left(p+\frac{1}{2}\right)^2+\frac{15}{4}$	**M1**	1.1b	TBC
	Deduces that det**M** ⩾ 0 for all values of p, therefore **M** always has an inverse	**A1**	2.2a	
		(2)		
2c	$\mathbf{M}^{-1} = \frac{1}{p^2+p+4}\begin{pmatrix} p+1 & 4 \\ -1 & p \end{pmatrix}$	**M1**	3.1a	TBC
	Uses any term from **M** + 10**M**$^{-1}$ = 5**I** Writes an equation in terms of p; for example, $-4+10\left(\frac{4}{p^2+p+4}\right)=0$	**M1**	1.1b	
	Solves to find p, obtaining $p = 2$	**A1**	2.3	
		(3)		
				(7 marks)

Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3a	Attempts to substitute $z = 1 - 3\text{i}$ into $\text{g}(z) = z^3 + pz^2 + 14z - 20$; for example, $(1 - 3\text{i})^3 + p(1 - 3\text{i})^2 + 14(1 - 3\text{i}) - 20 = 0$ is seen	**M1**	1.1a	TBC
	Attempts to expand the brackets; for example, $1 - 9\text{i} + 27\text{i}^2 - 27\text{i}^3 - 8p - 6p\text{i} + 14 - 42\text{i} - 20 = 0$ is seen	**M1**	1.1b	
	Equates either real or imaginary parts to find that $p = -4$	**A1**	2.2a	
		(3)		
3b	Demonstrates an understanding that if $z = 1 - 3\text{i}$ is a root, then $z = 1 + 3\text{i}$ is also a root; for example, $(z - (1 + 3\text{i}))(z - (1 - 3\text{i}))$ is seen	**M1**	1.2	TBC
	Deduces that $(az + b)(z^2 - 2z + 10) = z^3 - 4z^2 + 14z - 20$	**M1**	1.1b	
	Finds the root is $z = 2$, and states that the other two roots are $z = 2$ and $z = 1 + 3\text{i}$	**A1**	2.1	
		(3)		
				(6 marks)

Notes

3a: Award full marks for a correct answer obtained by substituting $z = 1 + 3\text{i}$.

3b: Award final accuracy mark providing $z = 2$ is stated clearly and $z = 1 + 3\text{i}$ is stated at some point during the question.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4	States the two rotation matrices: Reflection in the line $y = x$: $P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Rotation 90° anticlockwise: $Q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	**B1**	1.1b	TBC
	Sets up the correct matrix multiplication: $QP = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	**M1**	3.1a	
	Correctly finds $QP = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and states that this matrix represents a reflection in the *y*-axis	**B1**	2.4	
		(3)		
				(3 marks)

Notes

Must explain their final step clearly

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	States $\overrightarrow{AB}=(-5,9,-3)$ and $\overrightarrow{AC}=(1,0,-5)$	**M1**	1.1b	TBC
	Finds $\overrightarrow{AB}.\overrightarrow{AC}=-5\times1+9\times0+-3\times-5=10$	**A1**	1.1b	
		(2)		
5b	Finds $\left\|\overrightarrow{AB}\right\|=\sqrt{(-5)^2+9^2+(-3)^2}=\sqrt{115}$ and $\left\|\overrightarrow{AC}\right\|=\sqrt{1^2+0^2+(-5)^2}=\sqrt{26}$	**M1**	1.1b	TBC
	Makes an attempt to find the $\angle BAC$; for example, $\cos(\angle BAC)=\frac{\overrightarrow{AB}.\overrightarrow{AC}}{\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{AC}\right\|}=\frac{10}{(\sqrt{115})(\sqrt{26})}$ is seen	**M1**	1.1b	
	Finds the $\angle BAC=79.46...^\circ$	**A1**	1.1b	
	Makes an attempt to find the area of the triangle *ABC*; for example: area $=\frac{1}{2}\left\|\overrightarrow{AB}\right\|\left\|\overrightarrow{AC}\right\|\sin(\angle BAC)$ $=\frac{1}{2}(\sqrt{115})(\sqrt{26})\sin(79.46...^\circ)$ is seen	**M1**	1.1b	
	Correctly finds area = 26.9 (units^2)	**A1**	1.1b	
		(5)		
				(7 marks)

Notes

Exact working gives $\sin\theta=\frac{17\sqrt{10}}{\sqrt{115}\sqrt{26}}$ gving area = $\frac{17\sqrt{10}}{2}$

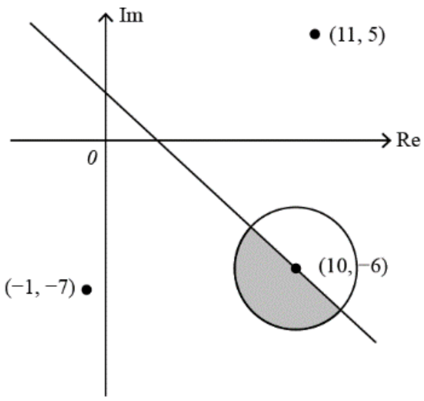
Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
6a	Substitutes at least one of the standard formulae into the expanded expression	**M1**	2.1	TBC
	Correctly finds $\sum_{r=1}^{n} r^2(2r-3) = 2\sum_{r=1}^{n} r^3 - 3\sum_{r=1}^{n} r^2$ $= 2\left(\frac{1}{4}\right)n^2(n+1)^2 - 3\left(\frac{1}{6}\right)n(n+1)(2n+1)$	**A1**	1.1b	
	Uses the common factor $n(n+1)$: $\frac{1}{2}n(n+1)\left[n(n+1)-(2n+1)\right]$	**M1**	1.1b	
	Obtains $\frac{1}{2}n(n+1)(n^2-n-1)$ showing all work clearly cso	**A1**	1.1b	
		(4)		
6b	Correctly finds $\sum_{r=1}^{n} 29r = 29\sum_{r=1}^{n} r = 29\left(\frac{1}{2}\right)n(n+1)$	**M1**	1.1b	TBC
	Equates $\frac{1}{2}n(n+1)(n^2-n-1)$ and $\frac{29}{2}n(n+1)$	**M1**	1.1b	
	Makes an attempt to solve to find n; for example, $n^2-n-30=0$ is seen	**M1**	1.1b	
	States that $n = 6$ as n must be a positive integer	**A1**	2.3	
		(4)		
				(8 marks)

Notes

6a: Award second method mark providing $n(n + 1)$ is factored out of the expression. Candidate does not need to factor the $\frac{1}{2}$ at this point in order to award the method mark.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	$y = ax^2 + 20 \Rightarrow 22 = a(10)^2 + 20$	**B1**	1.1b	TBC
	So, $a = \frac{2}{100} = \frac{1}{50}$			
		(1)		
7b	$V = \pi\int_{20}^{22} x^2 \text{ d}y = \pi\int_{20}^{22} 50y - 1000 \text{ d}y$	**M1**	3.3	
	Integrates to obtain $V = \pi\left[25y^2 - 1000y\right]_{20}^{22}$	**A1**	1.1b	
	Finds that the volume is 100π	**A1**	1.1b	
	Finds the volume of the cylinder is $\pi \times 10^2 \times 22 = 2200\pi$	**B1**	1.1b	
	Demonstrates an understanding of the need to subtract; for example, $2200\pi - 100\pi$ is seen	**M1**	3.4	
	Writes 2100π (cm^3)	**A1**	1.1b	
		(6)		
7c	The shape of the trophy display is unlikely to exactly follow the curve	**B1**	3.5b	
		(1)		
				(8 marks)

Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
8a	 Circle drawn	**M1**	1.1b	TBC
	Centre of circle labelled at (10, −6) and circle does not intersect the real axis	**A1**	1.1b	
	Makes attempt to draw on line that bisects the points (−1, −7) and (11, 5)	**M1**	1.1b	
	The line goes through the centre of the circle and intersects the positive real and positive imaginary axis	**A1**	3.1a	
	Shades in the region below the line and inside the circle (allow ft)	**M1**	3.1a	
	Fully correct solution	**A1**	1.1b	
		(6)		
8b	States that one solution is $\theta = -\frac{\pi}{2}$	**B1**	1.1b	TBC
	Attempts to use trigonometry to find the second solution; for example, $\arctan\left(\frac{5}{10}\right)$ or $\arcsin\left(\frac{1}{\sqrt{5}}\right)$ is seen	**M1**	3.1a	
	Correctly finds the second solution, $\theta = -\left(\frac{\pi}{2} - 2\times\arctan\left(\frac{5}{10}\right)\right) = -0.6435 = -0.64$ to 2 dp	**A1**	1.1b	
		(3)		
				(9 marks)

Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
9	States $w = 2x + 1 \Rightarrow x = \frac{w-1}{2}$	**B1**	3.1a	TBC
	States $\left(\frac{w-1}{2}\right)^3 - 4\left(\frac{w-1}{2}\right)^2 - 6\left(\frac{w-1}{2}\right) + 3 = 0$	**M1**	3.1a	
	Makes an attempt to manipulate the equation into the form $aw^3 + bw^2 + cw + d = 0$	**M1**	1.1b	
	At least two of p, q or r are correct	**A1**	1.1b	
	Fully correct final equation, $w^3 - 11w^2 - 5w + 39 = 0$ Or states, $p = -11$, $q = -5$ and $r = 39$	**A1**	1.1b	
		(5)		
				(5 marks)

Notes

Alternative method for first three marks

B1 $\alpha + \beta + \gamma = 4$, $\alpha\beta + \beta\gamma + \gamma\alpha = -6$ and $\alpha\beta\gamma = -3$

M1 Sum of roots,

$$(2\alpha + 1) + (2\beta + 1) + (2\gamma + 1)$$
$$= 2(\alpha + \beta + \gamma) + 3$$
$$= 2(4) + 3 = 11$$

Pair sum,

$$(2\alpha + 1)(2\beta + 1) + (2\beta + 1)(2\gamma + 1) + (2\gamma + 1)(2\alpha + 1)$$
$$= 4(\alpha\beta + \beta\gamma + \gamma\alpha) + 4(\alpha + \beta + \gamma) + 3$$
$$= 4(-6) + 4(4) + 3 = -5$$

Product,

$$(2\alpha + 1)(2\beta + 1)(2\gamma + 1)$$
$$= 8(\alpha\beta\gamma) + 4(\alpha\beta + \beta\gamma + \gamma\alpha) + 2(\alpha + \beta + \gamma) + 1$$
$$= 8(-3) + 4(-6) + 2(4) + 1 = -39$$

M1 Applies w^3 – (their sum roots)w^2 + (their pair sum)w – their product = 0

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
10	Makes an attempt to set up three equations in three unknowns Let x be the number of adult males, y the number of adult females and z the number of children.	**M1**	3.1b	TBC
	Correctly states at least **two** of the following $x + y + z = 4000$ $x + 200 = y$ $0.035x + 0.025y - 0.035z = 80$ $1.035x + 1.025y + 0.965z = 4080$ where x = the number of adult males, y = the number of adult females, z = the number of children	**A1**	1.1b	
	Attempts to set up a matrix equation	**M1**	3.1a	
	Correctly sets up a matrix equation; either, $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.035 & 0.025 & -0.035 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4000 \\ -200 \\ 80 \end{pmatrix}$ OR, $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1.035 & 1.025 & 0.965 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4000 \\ -200 \\ 4080 \end{pmatrix}$	**A1**	1.1b	
	Makes an attempt to use the inverse of the matrix from the line above; for example, $\begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.035 & 0.025 & -0.035 \end{pmatrix}^{-1}\begin{pmatrix} 4000 \\ -200 \\ 80 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$	**M1**	1.1b	
	Finds $x = 1600$, $y = 1800$ and $z = 600$	**A1**	1.1b	
	There were 1600 adult males, 1800 adult females and 600 children at the beginning of 2015	**A1 ft**	3.2a	
		(7)		
				(7 marks)

Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
11	Let $\mathrm{f}(n)=5^{n+1}+6^{2n-1}$ where $n \in Z^{+}$ Therefore, $\mathrm{f}(1)=5^2+6=31$; 31 is divisible by 31	**B1**	2.1	TBC
	Assume statement is true for $n = k$ So, assume $\mathrm{f}(k)=5^{k+1}+6^{2k-1}$ is divisible by 31	**M1**	2.4	
	Begins to build an expression for $n = k + 1$, $\mathrm{f}(k+1)=5^{(k+1)+1}+6^{2(k+1)-1}$ $=5^{k+2}+6^{2k+1}$	**M1**	2.1	
	Recognises the need to find $\mathrm{f}(k+1)-\mathrm{f}(k)$ and simplifies, $\mathrm{f}(k+1)-\mathrm{f}(k)=5^{k+2}+6^{2k+1}-\left(5^{k+1}+6^{2k-1}\right)$ $=5^{k+1}(5-1)+6^{2k-1}\left(6^2-1\right)$ $=4\left(5^{k+1}\right)+35\left(6^{2k-1}\right)$ $=4\left(5^{k+1}\right)+4\left(6^{2k-1}\right)+31\left(6^{2k-1}\right)$ $=4\left(5^{k+1}+6^{2k-1}\right)+31\left(6^{2k-1}\right)$	**M1**	3.1a	
	Use expression for $\mathrm{f}(k+1)-\mathrm{f}(k)$ to find an expression for $\mathrm{f}(k+1)$ $\mathrm{f}(k+1)=\mathrm{f}(k)+4\left(5^{k+1}+6^{2k-1}\right)+31\left(6^{2k-1}\right)$ $\mathrm{f}(k+1)=\mathrm{f}(k)+4\mathrm{f}(k)+31\left(6^{2k-1}\right)$ $\mathrm{f}(k+1)=5\mathrm{f}(k)+31\left(6^{2k-1}\right)$	**A1**	1.1b	
	Therefore, f(n) is divisible by 31 when $n = k + 1$	**B1**	2.2a	
	Demonstrates an understanding of the process of mathematical induction: If f(n) is divisible by 31 when $n = k$, then it has been shown that f(n) is also divisible by 31 when $n = k + 1$ As f(n) is divisible by 31 when $n = 1$, f(n) is also divisible by 31 for all $n \geqslant 1$ and $n \in Z^{+}$ by mathematical induction	**A1**	2.4	
		(7)		
				(7 marks)

Notes

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
12a	Finds $\overrightarrow{AB} = (-5, 5, -2)$	**M1**	3.1a	TBC
	States that a general point, P, on $\overrightarrow{AB}$ is $P = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix}$	**M1**	1.1b	
	At the shortest (perpendicular) distance $\begin{pmatrix} -3-5\lambda \\ 1+5\lambda \\ 7-2\lambda \end{pmatrix} \begin{pmatrix} -5 \\ 5 \\ -2 \end{pmatrix} = 0$	**M1**	1.1b	
	Solves $15+25\lambda+5+25\lambda-14+4\lambda=0$ to obtain $\lambda = -\frac{1}{9}$	**A1**	1.1b	
	Substitutes λ to find the coordinates of P: $P\begin{pmatrix} -\frac{22}{9} \\ \frac{4}{9} \\ \frac{65}{9} \end{pmatrix}$ and attempts to find the distance $OP = \sqrt{\left(-\frac{22}{9}\right)^2 + \left(\frac{4}{9}\right)^2 + \left(\frac{65}{9}\right)^2}$	**M1**	3.1a	
	Finds minimum distance = $\sqrt{\frac{4725}{81}} \approx 7.64$ km	**A1**	1.1b	
	Concludes that, as this is less than 8 km, the plane cannot move from A to B in a straight line without entering the 'no-fly' zone	**A1 ft**	3.2a	
		(7)		
12b	Aeroplane is unlikely to move in exactly a straight line o.e.	**B1**	3.5b	TBC
		(1)		
				(8 marks)

Notes