| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| 1 | Finds $\det\mathbf{M} = (2)(3) - (-p)(p+4) = p^2 + 4p + 6$ | M1 | 1.1b | TBC |
| | Completes the square to show $p^2 + 4p + 6 = (p+2)^2 + 2$ | M1 | 2.2a | |
| | Concludes that $(p + 2)^2 + 2 > 0$ for all values of $p$. Therefore $\det \mathbf{M} \neq 0$ and $\mathbf{M}$ is non-singular. | B1 | 3.2a | |
| | | | | **(3 marks)** |

**Notes**

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| 2 | Finds $\mathbf{M}^2 = \begin{pmatrix} a & 21 \\ b & -8 \end{pmatrix}\begin{pmatrix} a & 21 \\ b & -8 \end{pmatrix} = \begin{pmatrix} a^2+21b & 21a-168 \\ ab-8b & 21b+64 \end{pmatrix}$ | M1 | 1.1b | TBC |
| | Deduces that $21a - 168 = 0$ and solves to find $a = 8$ | A1* | 2.2a | |
| | Deduces that $a^2 + 21b = 1$ and solves to find $b = -3$ | A1* | 2.2a | |
| | | | | **(3 marks)** |

**Notes**

**2**

Can use any of the following equations to find $a$ and $b$. Award 1 mark for finding $a$ and 1 mark for finding $b$.

$a^2 + 21b = 1$

$21a - 168 = 0$

$ab - 8b = 0$

$21b + 64 = 1$

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| **3a** | States either $\cos\theta = -\frac{\sqrt{3}}{2}$ or $\sin\theta = \frac{1}{2}$ or $\tan\theta = -\frac{1}{\sqrt{3}}$ | **M1** | 1.1b | TBC |
| | Finds $\theta$ = 150° and concludes this is a rotation of 150° anticlockwise about the origin. | **B1** | 3.2a | |
| | | **(2)** | | |
| **3b** | Sets up a matrix equation of the form: $\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$ or two separate equations of the form $\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ | **M1** | 1.1a | TBC |
| | Finds $\begin{pmatrix} -\frac{\sqrt{3}}{2}-1 & -\sqrt{3}-\frac{5}{2} \\ \frac{1}{2}-\sqrt{3} & 1-\frac{5\sqrt{3}}{2} \end{pmatrix}$ or $\begin{pmatrix} -\frac{\sqrt{3}}{2}-1 \\ \frac{1}{2}-\sqrt{3} \end{pmatrix}$ and $\begin{pmatrix} -\sqrt{3}-\frac{5}{2} \\ 1-\frac{5\sqrt{3}}{2} \end{pmatrix}$ | **M1** | 1.1b | |
| | States $P'\left(\frac{-\sqrt{3}-2}{2}, \frac{1-2\sqrt{3}}{2}\right)$ and $Q'\left(\frac{-2\sqrt{3}-5}{2}, \frac{2-5\sqrt{3}}{2}\right)$ | **A1** | 3.2a | |
| | | **(3)** | | |

| | | | | |
|---|---|---|---|---|
| **3c** | Finds $\mathbf{M}^2 = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ | **M1** | 1.1b | TBC |
| | States either $\cos\theta = \frac{1}{2}$ or $\sin\theta = -\frac{\sqrt{3}}{2}$ or $\tan\theta = -\sqrt{3}$ | **M1** | 1.1b | |
| | Finds $\theta = 300°$ and concludes this is a rotation of 300° anticlockwise about the origin.<br>or<br>Finds $\theta = -60°$ and concludes that it is a rotation of 60° clockwise about the orgin. | **B1** | 3.2a | |
| | | **(3)** | | |
| | | | | **(8 marks)** |

**Notes**

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| 4a | Reflection in the line $y = -x$ | B1 | 3.2a | TBC |
| | | (1) | | |
| 4b | Calculates $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -b \\ -a \end{pmatrix}$ | M1 | 1.1b | TBC |
| | States or implies $-b = 4 + 2a$ and $-b = 4 + 2a = 5 + b$ | M1 | 2.2a | |
| | Finds $a = 1$ and $b = -6$ | A1 | 1.1b | |
| | | (3) | | |
| | | | | (4 marks) |

**Notes**

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| **5** | Finds $\mathbf{P}^2 = \begin{pmatrix} q^2+5 & 0 \\ 0 & q^2+5 \end{pmatrix}$ | **M1** | 1.1b | TBC |
| | States that this is an enlargement. | **A1** | 3.2a | |
| | States scale factor is $q^2 + 5$ and centre is (0, 0). | **A1** | 3.2a | |
| | | | | **(3 marks)** |

**Notes**

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| 6a | Writes the matrix representing a reflection in the plane $y = 0$: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | B1 | 3.1a | TBC |
| | | (1) | | |
| 6b | Finds the midpoint of the line segment = (5, 5, 9) | M1 | 1.1b | TBC |
| | Makes an attempt to calculate $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 5 \\ 9 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$<br>Minimum required is setting up the calculation. | M1 | 2.2a | |
| | Correctly finds the coordinates (5, −5, 9) | A1 | 3.1b | |
| | | (3) | | |
| 6c | States or implies that **N** is the inverse of **M**. | M1 | 2.2a | TBC |
| | Finds $\mathbf{N} = \mathbf{M}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ | A1 | 1.1b | |
| | | (2) | | |
| | | | | **(6 marks)** |

**Notes**

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| 7a | $\det(\mathbf{M}) = (-1)(-1) - (\sqrt{2})(-\sqrt{2})$ | M1 | 1.1a | TBC |
| | **M** is non-singular because det (**M**) = 3 and so det (**M**) ≠ 0 | A1 | 2.4 | |
| | | (2) | | |
| 7b | Area ($S$) = 3 × 20 = 60 | B1 ft | 1.2 | TBC |
| | | (1) | | |
| 7c | Shows $k = \sqrt{\det(\mathbf{M})} = \sqrt{(-1)(-1) - (\sqrt{2})(-\sqrt{2})}$ | M1 | 1.1b | TBC |
| | States $k = \sqrt{3}$ | A1 ft | 1.1b | |
| | | (2) | | |
| 7d | States $\cos\theta = -\frac{1}{\sqrt{3}}$ or $\sin\theta = \sqrt{\frac{2}{3}}$ or $\tan\theta = -\sqrt{2}$ | M1 | 1.1b | TBC |
| | $\theta$ = 125.3° Accept answers which round to 125.3° | A1 | 1.1b | |
| | | (2) | | |
| | **(7 marks)** | | | |
| | **Notes** | | | |

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| 8a | States either $\cos\theta = -\frac{1}{2}$ or $\sin\theta = -\frac{\sqrt{3}}{2}$ or $\tan\theta = \sqrt{3}$ | M1 | 1.1b | TBC |
| | Finds $\theta = 240°$ and concludes this is a rotation of 240° anticlockwise about the $z$-axis. | B1 | 3.2a | |
| | | (2) | | |
| 8b | Makes an attempt to calculate $\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -p \\ 0 \\ p \end{pmatrix}$<br>Minimum required is setting up the calculation. | M1 | 1.1b | TBC |
| | Correctly finds $\begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -p \\ 0 \\ p \end{pmatrix} = \begin{pmatrix} \frac{1}{2}p \\ \frac{\sqrt{3}}{2}p \\ p \end{pmatrix}$ | A1 | 1.1b | |
| | | (2) | | |
| | | | | **(4 marks)** |

**Notes**

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| **9** | Attempts to set up three equations with three unknowns. | **M1** | 3.1b | TBC |
| | At least two equations are correct, with variables defined.<br>$x$ = area of residential land<br>$y$ = area of commercial land<br>$z$ = area of recreational land<br>$x+y+z=200$<br>$-y+z=20$<br>$1.22x+0.9y+1.28z=240$ | **A1** | 1.1b | |
| | Sets up a matrix equation of the form, $\begin{pmatrix} \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} \ldots \\ \ldots \\ \ldots \end{pmatrix}$,<br>where '…' are numerical values. | **M1** | 3.1a | |
| | States the correct matrix equation: $\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1.22 & 0.9 & 1.28 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 200 \\ 20 \\ 240 \end{pmatrix}$ | **A1** | 1.1b | |
| | Attempts to use an inverse matrix to find the values of $x$, $y$ and $z$.<br>$\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1.22 & 0.9 & 1.28 \end{pmatrix}^{-1}\begin{pmatrix} 200 \\ 20 \\ 240 \end{pmatrix}$ | **M1** | 1.1b | |
| | Finds the correct answers for $x$, $y$ and $z$: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 140 \\ 20 \\ 40 \end{pmatrix}$ | **A1** | 1.1b | |
| | Puts their answer into context. In 2001, there were 140 square kilometres assigned to residential, 20 square kilometres assigned to commerical and 40 square kilometres assigned to recreation. | **A1 ft** | 3.2a | |
| | | | | **(7 marks)** |

**Notes**

**9**

Note the inverse matrix of $\begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ 1.22 & 0.9 & 1.28 \end{pmatrix}$ is $\frac{50}{13}\begin{pmatrix} -2.18 & -0.38 & 2 \\ 1.22 & 0.06 & -1 \\ 1.22 & 0.32 & -1 \end{pmatrix}$

| Q | Scheme | Marks | AOs | Pearson Progression Step and Progress descriptor |
|---|---|---|---|---|
| 10a | States either $\cos\theta = -\frac{1}{\sqrt{2}}$ or $\sin\theta = \frac{1}{\sqrt{2}}$ or $\tan\theta = -1$ | M1 | 1.1b | TBC |
| | Finds $\theta$ = 135° and concludes this is a rotation of 135° anticlockwise about the origin.<br>or<br>Finds $\theta$ = −45° and concludes this is a rotation of 45° clockwise about the origin. | B1 | 3.2a | |
| | | (2) | | |
| 10b | Finds $\mathbf{M}^{-1} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$ | M1 | 1.1b | TBC |
| | States or implies that if $\mathbf{M}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$ then $\begin{pmatrix} a \\ b \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} -\sqrt{2} \\ 3\sqrt{2} \end{pmatrix}$ | M1 | 3.1a | |
| | Correctly solves to find $a$ = 4 and $b$ = −2 | A1 | 1.1b | |
| | | (3) | | |
| | | | | **(5 marks)** |

**Notes**