Jun 07

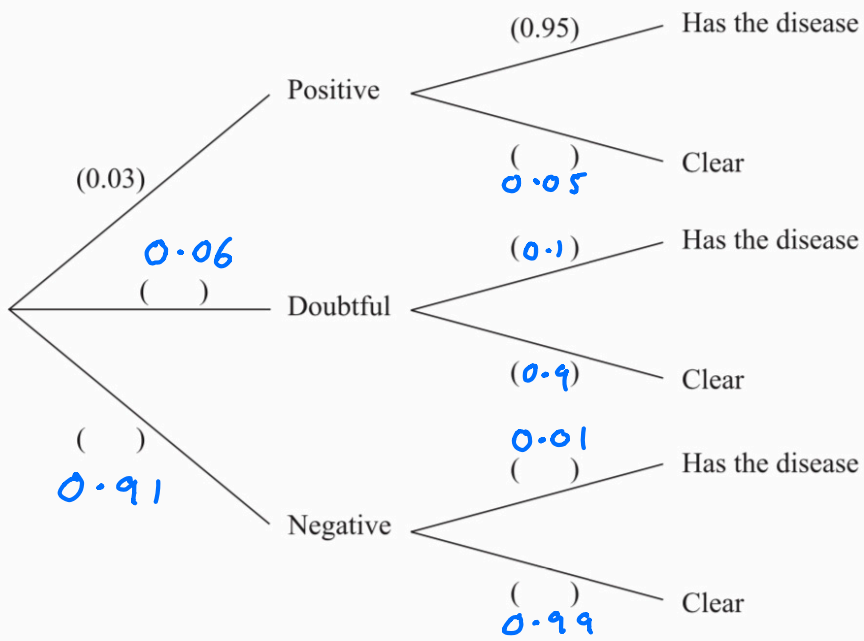
Section B (36 marks)

7 A screening test for a particular disease is applied to everyone in a large population. The test classifies people into three groups: 'positive', 'doubtful' and 'negative'. Of the population, 3% is classified as positive, 6% as doubtful and the rest negative.

In fact, of the people who test positive, only 95% have the disease. Of the people who test doubtful, 10% have the disease. Of the people who test negative, 1% actually have the disease.

People who do not have the disease are described as 'clear'.

(i) Copy and complete the tree diagram to show this information. [4]



(ii) Find the probability that a randomly selected person tests negative and is clear. [2]

(iii) Find the probability that a randomly selected person has the disease. [3]

(iv) Find the probability that a randomly selected person tests negative **given** that the person has the disease. [3]

(v) Comment briefly on what your answer to part **(iv)** indicates about the effectiveness of the screening test. [2]

Once the test has been carried out, those people who test doubtful are given a detailed medical examination. If a person has the disease the examination will correctly identify this in 98% of cases. If a person is clear, the examination will always correctly identify this.

(vi) A person is selected at random. Find the probability that this person either tests negative originally or tests doubtful and is then cleared in the detailed medical examination. [4]

ii) $P(\text{Neg} \cap \text{Clear}) = 0.91 \times 0.99 = 0.9009$

iii) $P(\text{Has Disease}) = 0.03 \times 0.95 + 0.06 \times 0.1 + 0.91 \times 0.01$

$= 0.0436$

iv) $P(\text{Neg} / \text{Has Disease}) = \dfrac{P(\text{Neg} \cap \text{Has Disease})}{P(\text{Has Disease})}$

$$= \frac{0.91 \times 0.01}{0.0436} = \frac{91}{436} = 0.2087$$

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v) 21% of people with the disease will test negative. This is an uncomfortably high proportion

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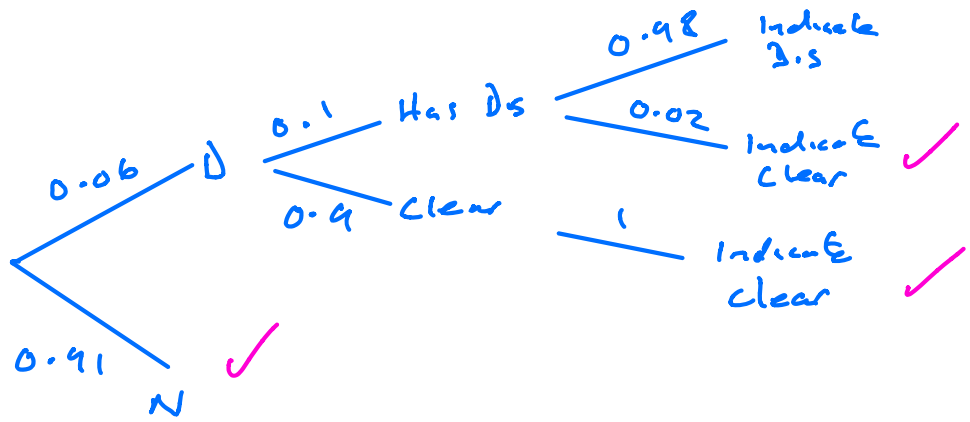
vi)



$$0.91 + 0.06 \times 0.9 \times 1 + 0.06 \times 0.1 \times 0.02$$

$$= 0.96412$$

Jun 07

8 A multinational accountancy firm receives a large number of job applications from graduates each year. On average 20% of applicants are successful.

A researcher in the human resources department of the firm selects a random sample of 17 graduate applicants.

(i) Find the probability that at least 4 of the 17 applicants are successful. [3]

(ii) Find the expected number of successful applicants in the sample. [2]

(iii) Find the most likely number of successful applicants in the sample, justifying your answer. [3]

It is suggested that mathematics graduates are more likely to be successful than those from other fields. In order to test this suggestion, the researcher decides to select a new random sample of 17 mathematics graduate applicants. The researcher then carries out a hypothesis test at the 5% significance level.

(iv) (*A*) Write down suitable null and alternative hypotheses for the test.

(*B*) Give a reason for your choice of the alternative hypothesis. [4]

(v) Find the critical region for the test at the 5% level, showing all of your calculations. [4]

(vi) Explain why the critical region found in part **(v)** would be unaltered if a 10% significance level were used. [2]

i) $X \sim B(\overset{n}{17}, \overset{p}{0.2})$

$P(X \geq 4) = 1 - P(X \leq 3)$

$= 1 - 0.5489$

$= 0.4511$

ii) $E(X) = 17 \times 0.2 = 3.4$

iii) $P(X=2) = 0.1914$

$P(X=3) = 0.2392$

$P(X=4) = 0.2093$

Most likely successful $= 3$

Bin probs increase then decrease

iv) $H_0 : p = 0.2$ $\quad$ p = prob of maths grad being successful

$H_1 : p > 0.2$

$H_1 : p > 0.2$ because suspected maths grads more successful

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v) $X \sim B(17, 0.2)$

$P(X \geq 6) = 1 - P(X \leq 5) = 1 - 0.8942 = 0.1058 > 5\%$

$P(X \geq 7) = 1 - P(X \leq 6) = 1 - 0.9623 = 0.0377 < 5\%$

CR $\quad X \geq 7$ $\quad$ or $\{7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17\}$

vi) Since $P(X \geq 6) > 10\%$

6 would not be in CR for 10% test

so CR would still be $X \geq 7$

HWK QUESTION BELOW

JAN 2009

7 An online shopping company takes orders through its website. On average 80% of orders from the website are delivered within 24 hours. The quality controller selects 10 orders at random to check when they are delivered.

(i) Find the probability that

(A) exactly 8 of these orders are delivered within 24 hours, **[3]**

(B) at least 8 of these orders are delivered within 24 hours. **[2]**

The company changes its delivery method. The quality controller suspects that the changes will mean that fewer than 80% of orders will be delivered within 24 hours. A random sample of 18 orders is checked and it is found that 12 of them arrive within 24 hours.

(ii) Write down suitable hypotheses and carry out a test at the 5% significance level to determine whether there is any evidence to support the quality controller's suspicion. **[7]**

(iii) A statistician argues that it is possible that the new method could result in either better or worse delivery times. Therefore it would be better to carry out a 2-tail test at the 5% significance level. State the alternative hypothesis for this test. Assuming that the sample size is still 18, find the critical region for this test, showing all of your calculations. **[7]**