Mixed Exercise 12

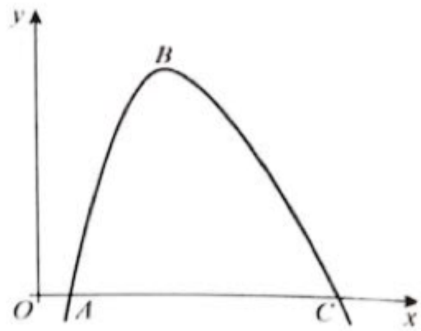
E **21** The diagram shows part of the curve with equation $y = \mathrm{f}(x)$, where:

$$\mathrm{f}(x) = 200 - \frac{250}{x} - x^2, x > 0$$

The curve cuts the x-axis at the points A and C.
The point B is the maximum point of the curve.

a Find $\mathrm{f}'(x)$. **(3 marks)**

b Use your answer to part **a** to calculate the coordinates of B. **(4 marks)**



a) $f(x) = 200 - 250x^{-1} - x^2$

$f'(x) = 250x^{-2} - 2x$

$f'(x) = \frac{250}{x^2} - 2x$

b) At t.p. B $f'(x) = 0$

$\Rightarrow \frac{250}{x^2} - 2x = 0$

$\frac{250}{x^2} = 2x$

$250 = 2x^3$

$125 = x^3$

$\Rightarrow x = 5$

$f(5) = 200 - \frac{250}{5} - 5^2$

$= 200 - 50 - 25$

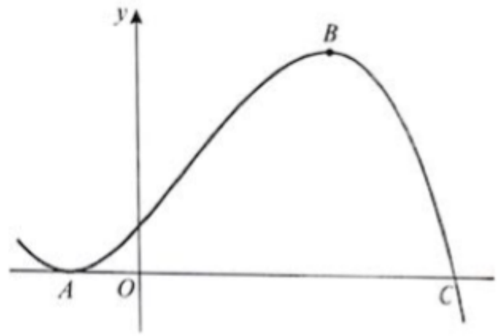
$= 125$

$\therefore B(5, 125)$

(E) **23** The diagram shows part of the curve with equation $y = 3 + 5x + x^2 - x^3$. The curve touches the x-axis at A and crosses the x-axis at C. The points A and B are stationary points on the curve.

a Show that C has coordinates (3, 0). **(1 mark)**

b Using calculus and showing all your working, find the coordinates of A and B. **(5 marks)**



a) when $x = 3$ $\quad y = 3 + 5(3) + 3^2 - 3^3$

$= 3 + 15 + 9 - 27$

$= 0$

$\therefore C(3, 0)$

b) $y = 3 + 5x + x^2 - x^3$

$\frac{dy}{dx} = 5 + 2x - 3x^2$

At st pt $\frac{dy}{dx} = 0$

$\Rightarrow 5 + 2x - 3x^2 = 0$

$\Rightarrow 3x^2 - 2x - 5 = 0$

$(3x - 5)(x + 1) = 0$

$\Rightarrow x = \frac{5}{3}$ or $x = -1$

when $x = -1$ $\quad y = 3 + 5(-1) + (-1)^2 - (-1)^3$

$y = 3 - 5 + 1 + 1 = 0$

$\therefore A(-1, 0)$

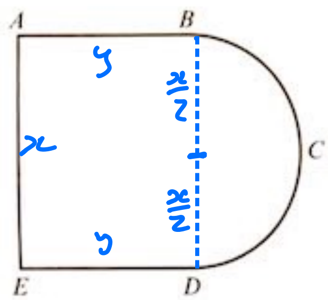
when $x = \frac{5}{3}$ $\quad y = 3 + 5\left(\frac{5}{3}\right) + \left(\frac{5}{3}\right)^2 - \left(\frac{5}{3}\right)^3$

$$y = 3 + \frac{25}{3} + \frac{25}{9} - \frac{125}{27}$$

$$y = \frac{256}{27}$$

$$\therefore \quad B\left(\frac{5}{3}, \frac{256}{27}\right)$$

P **27** A wire is bent into the plane shape $ABCDE$ as shown. Shape $ABDE$ is a rectangle and BCD is a semicircle with diameter BD. The area of the region enclosed by the wire is $R\,\text{m}^2$, $AE = x$ metres, and $AB = ED = y$ metres. The total length of the wire is 2 m.



a Find an expression for y in terms of x. **(3 marks)**

b Prove that $R = \frac{x}{8}(8 - 4x - \pi x)$. **(4 marks)**

Given that x can vary, using calculus and showing your working:

c find the maximum value of R. (You do not have to prove that the value you obtain is a maximum.) **(5 marks)**

a) Semi-circle length $= \pi r = \frac{\pi x}{2}$

$$\therefore \quad x + 2y + \frac{\pi x}{2} = 2$$

$$2y = 2 - x - \frac{\pi x}{2}$$

$$y = 1 - \frac{x}{2} - \frac{\pi x}{4}$$

b) $R =$ Area of rectangle + Area of semi-circle

$$R = xy + \frac{\pi\left(\frac{x}{2}\right)^2}{2}$$

$$R = x\left(1 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{\pi x^2}{8}$$

$$R = \frac{x}{8}\left(8 - 4x - 2\pi x\right) + \frac{x}{8}\left(\pi x\right)$$

$$R = \frac{x}{8}\left(8 - 4x - 2\pi x + \pi x\right)$$

$$R = \frac{x}{8}\left(8 - 4x - \pi x\right)$$

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c)

$$R = x - \frac{x^2}{2} - \frac{\pi x^2}{8}$$

$$\frac{dR}{dx} = 1 - x - \frac{\pi x}{4}$$

At st pt $\frac{dR}{dx} = 0 \Rightarrow 1 - x - \frac{\pi x}{4} = 0$

$$\Rightarrow 1 = x + \frac{\pi x}{4}$$

$$1 = x\left(1 + \frac{\pi}{4}\right)$$

$$\frac{1}{\left(1 + \frac{\pi}{4}\right)} = x$$

$$x = 0.560099$$

$$x = 0.56$$

$$R = 0.56 - \frac{0.56^2}{2} - \frac{\pi \times 0.56^2}{8}$$

$$R = 0.28 \text{ m}^2$$

CLASSWORK

Mixed Exercise 12

Q22, Q24, Q26, Q28, Q29, Q1
