Algebraic Fractions 2

4 Simplify each of these.

a $\frac{x}{2} \times \frac{x}{3}$

b $\frac{2x}{7} \times \frac{3y}{4}$

c $\frac{4x}{3y} \times \frac{2y}{x}$

d $\frac{4y^2}{9x} \times \frac{3x^2}{2y}$

e $\frac{x}{2} \times \frac{x-2}{5}$

f $\frac{x-3}{15} \times \frac{5}{2x-6}$

g $\frac{2x+1}{2} \times \frac{3x+1}{4}$

h $\frac{x}{5} \times \frac{2x+1}{3}$

i $\frac{x-2}{2} \times \frac{4}{x-3}$

j $\frac{x-5}{10} \times \frac{5}{x^2-5x}$

5 Simplify each of these.

a $\frac{x}{2} \div \frac{x}{3}$

b $\frac{2x}{7} \div \frac{4y}{14}$

c $\frac{4x}{3y} \div \frac{x}{2y}$

d $\frac{4y^2}{9x} \div \frac{2y}{3x^2}$

e $\frac{x}{2} \div \frac{x-2}{5}$

f $\frac{x-3}{15} \div \frac{5}{2x-6}$

g $\frac{2x+1}{2} \div \frac{4x+2}{4}$

h $\frac{x}{6} \div \frac{2x^2+x}{3}$

i $\frac{x-2}{12} \div \frac{4}{x-3}$

j $\frac{x-5}{10} \div \frac{x^2-5x}{5}$

6 Simplify each of these. Factorise and cancel where appropriate.

a $\frac{3x}{4} + \frac{x}{4}$

b $\frac{3x}{4} - \frac{x}{4}$

c $\frac{3x}{4} \times \frac{x}{4}$

d $\frac{3x}{4} \div \frac{x}{4}$

e $\frac{3x+1}{2} + \frac{x-2}{5}$

f $\frac{3x+1}{2} - \frac{x-2}{5}$

g $\frac{3x+1}{2} \times \frac{x-2}{5}$

h $\frac{x^2-9}{10} \times \frac{5}{x-3}$

i $\frac{2x+3}{5} \div \frac{6x+9}{10}$

j $\frac{2x^2}{9} - \frac{2y^2}{3}$

7 Show that each algebraic fraction simplifies to the given expression.

a $\frac{2}{x+1} + \frac{5}{x+2} = 3$ simplifies to $3x^2 + 2x - 3 = 0$

b $\frac{4}{x-2} + \frac{7}{x+1} = 3$ simplifies to $3x^2 - 14x + 4 = 0$

c $\frac{3}{4x+1} - \frac{4}{x+2} = 2$ simplifies to $8x^2 + 31x + 2 = 0$

d $\frac{2}{2x-1} - \frac{6}{x+1} = 11$ simplifies to $22x^2 + 21x - 19 = 0$

e $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$ simplifies to $x^2 - x = 0$

4 Simplify each of these.

a $\frac{x}{2} \times \frac{x}{3}$

b $\frac{2x}{7} \times \frac{3y}{4}$

c $\frac{4x}{3y} \times \frac{2y}{x}$

d $\frac{4y^2}{9x} \times \frac{3x^2}{2y}$

e $\frac{x}{2} \times \frac{x-2}{5}$

f $\frac{x-3}{15} \times \frac{5}{2x-6}$

g $\frac{2x+1}{2} \times \frac{3x+1}{4}$

h $\frac{x}{5} \times \frac{2x+1}{3}$

i $\frac{x-2}{2} \times \frac{4}{x-3}$

j $\frac{x-5}{10} \times \frac{5}{x^2-5x}$

4a) $\frac{x}{2} \times \frac{x}{3} = \frac{x^2}{6}$

4c) $\frac{4\cancel{x}}{3\cancel{y}} \times \frac{2\cancel{y}}{\cancel{x}} = \frac{8}{3}$

4e) $\frac{x}{2} \times \frac{(x-2)}{5} = \frac{x(x-2)}{2 \times 5} = \frac{x^2-2x}{10}$

4g) $\frac{(2x+1)}{2} \times \frac{(3x+1)}{4} = \frac{6x^2+3x+2x+1}{8}$

$= \frac{6x^2+5x+1}{8}$

4i) $\frac{(x-2)}{2} \times \frac{4}{(x-3)} = \frac{2(x-2)}{x-3}$ or $\frac{2x-4}{x-3}$

4f) $\frac{x-3}{15} \times \frac{5}{2x-6} = \frac{\cancel{(x-3)}}{\cancel{15}\,3} \times \frac{\cancel{5}^{1}}{2\cancel{(x-3)}} = \frac{1}{6}$

Classwork

4b) $\frac{2x}{7} \times \frac{3y}{4} = \frac{3xy}{14}$

4d) $\frac{4y^2}{9x} \times \frac{3x^2}{2y} = \frac{12xy}{18} = \frac{2xy}{3}$

4h) $\frac{x}{5} \times \frac{(2x+1)}{3} = \frac{2x^2+x}{15}$

4j) $\frac{x-5}{10} \times \frac{5}{x^2-5x} = \frac{\cancel{(x-5)}}{\cancel{10}_2} \times \frac{\cancel{5}^1}{x\cancel{(x-5)}}$

$= \frac{1}{2x}$

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6 Simplify each of these. Factorise and cancel where appropriate.

a $\frac{3x}{4} + \frac{x}{4}$

b $\frac{3x}{4} - \frac{x}{4}$

c $\frac{3x}{4} \times \frac{x}{4}$

d $\frac{3x}{4} \div \frac{x}{4}$

e $\frac{3x+1}{2} + \frac{x-2}{5}$

f $\frac{3x+1}{2} - \frac{x-2}{5}$

g $\frac{3x+1}{2} \times \frac{x-2}{5}$

h $\frac{x^2-9}{10} \times \frac{5}{x-3}$

i $\frac{2x+3}{5} \div \frac{6x+9}{10}$

j $\frac{2x^2}{9} - \frac{2y^2}{3}$

6a) $\frac{3x}{4} + \frac{x}{4} = \frac{3x+x}{4} = \frac{4x}{4} = x$

6h) $\frac{x^2-9}{10} \times \frac{5}{x-3} = \frac{(x+3)\cancel{(x-3)}}{\cancel{10}_2} \times \frac{\cancel{5}^1}{\cancel{(x-3)}}$

$= \frac{x+3}{2}$

Classwork 6b, 6f, 6i,

6b) $\frac{3x}{4} - \frac{x}{4} = \frac{3x-x}{4} = \frac{2x}{4} = \frac{x}{2}$

6f)

f $\frac{3x+1}{2} - \frac{x-2}{5}$ $= \frac{5(3x+1) - 2(x-2)}{10}$

$= \frac{15x+5-2x+4}{10} = \frac{13x+9}{10}$

6i)

i $\frac{2x+3}{5} \div \frac{6x+9}{10}$ $= \frac{2x+3}{5} \times \frac{10}{6x+9}$

$= \frac{\cancel{(2x+3)}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{3\cancel{(2x+3)}} = \frac{2}{3}$

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7 Show that each algebraic fraction simplifies to the given expression.

a $\frac{2}{x+1} + \frac{5}{x+2} = 3$ simplifies to $3x^2 + 2x - 3 = 0$

b $\frac{4}{x-2} + \frac{7}{x+1} = 3$ simplifies to $3x^2 - 14x + 4 = 0$

c $\frac{3}{4x+1} - \frac{4}{x+2} = 2$ simplifies to $8x^2 + 31x + 2 = 0$

d $\frac{2}{2x-1} - \frac{6}{x+1} = 11$ simplifies to $22x^2 + 21x - 19 = 0$

e $\frac{3}{2x-1} - \frac{4}{3x-1} = 1$ simplifies to $x^2 - x = 0$

7a) $\frac{2}{x+1} + \frac{5}{x+2} = 3$

$\frac{2\cancel{(x+1)}(x+2)}{\cancel{(x+1)}} + \frac{5(x+1)\cancel{(x+2)}}{\cancel{(x+2)}} = 3(x+1)(x+2)$

$2x+4 + 5x+5 = 3[x^2 + 3x + 2]$

$7x + 9 = 3x^2 + 9x + 6$

$$0 = 3x^2 + 9x + 6 - 7x - 9$$

$$0 = 3x^2 + 2x - 3$$

7e) $\dfrac{3}{(2x-1)} - \dfrac{4}{(3x-1)} = 1$

$$\frac{3\cancel{(2x-1)}(3x-1)}{\cancel{(2x-1)}} - \frac{4(2x-1)\cancel{(3x-1)}}{\cancel{(3x-1)}} = 1(2x-1)(3x-1)$$

$$9x - 3 - 8x + 4 = 6x^2 - 3x - 2x + 1$$

$$x + 1 = 6x^2 - 5x + 1$$

$$0 = 6x^2 - 5x + 1 - x - 1$$

$$0 = 6x^2 - 6x$$

$$0 = x^2 - x$$

Classwork

b $\dfrac{4}{x-2} + \dfrac{7}{x+1} = 3$ simplifies to $3x^2 - 14x + 4 = 0$

$$\frac{4\cancel{(x-2)}(x+1)}{\cancel{(x-2)}} + \frac{7(x-2)\cancel{(x+1)}}{\cancel{(x+1)}} = 3(x-2)(x+1)$$

$$4x + 4 + 7x - 14 = 3[x^2 - x - 2]$$

$$11x - 10 = 3x^2 - 3x - 6$$

$$0 = 3x^2 - 3x - 6 - 11x + 10$$

$$0 = 3x^2 - 14x + 4$$

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c $\dfrac{3}{4x+1} - \dfrac{4}{x+2} = 2$ simplifies to $8x^2 + 31x + 2 = 0$

$$\frac{3\cancel{(4x+1)}(x+2)}{\cancel{(4x+1)}} - \frac{4(4x+1)\cancel{(x+2)}}{\cancel{(x+2)}} = 2(4x+1)(x+2)$$

$$3x+6-16x-4 = 2\left[4x^2+9x+2\right]$$

$$-13x+2 = 8x^2+18x+4$$

$$0 = 8x^2+18x+4+13x-2$$

$$0 = 8x^2+31x+2$$

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d $\dfrac{2}{2x-1} - \dfrac{6}{x+1} = 11$ simplifies to $22x^2 + 21x - 19 = 0$

$$\frac{2\cancel{(2x-1)}(x+1)}{\cancel{(2x-1)}} - \frac{6(2x-1)\cancel{(x+1)}}{\cancel{(x+1)}} = 11(2x-1)(x+1)$$

$$2x+2-12x+6 = 11\left[2x^2+x-1\right]$$

$$-10x+8 = 22x^2+11x-11$$

$$0 = 22x^2+11x-11+10x-8$$

$$0 = 22x^2+21x-19$$
