Vectors in Geometry


$$
\begin{aligned}
\overrightarrow{B O} & =\overrightarrow{B A}+\overrightarrow{A O} \\
& =-\underline{a}+\underline{b}
\end{aligned}
$$

$$
\begin{array}{lll}
\overrightarrow{A B}=a & \overrightarrow{A O}=b & \overrightarrow{B O}=-\underline{a}+b \\
\overrightarrow{F O}=a & \overrightarrow{F E}=\underline{b} & \overrightarrow{C D}=-\underline{a}+\underline{b} \\
\overrightarrow{O C}=a & \overrightarrow{O D}=\underline{b} & \overrightarrow{O E}=-\underline{a}+\underline{b} \\
\overrightarrow{E D}=a & \overrightarrow{B C}=\underline{b} & \overrightarrow{A F}=-\underline{a}+\underline{b}
\end{array}
$$

Further Examples

$$
\begin{aligned}
\overrightarrow{A E} & =\overrightarrow{A F}+\overrightarrow{F E} \\
& =-\underline{a}+\underline{b}+\underline{b}=2 \underline{b}-\underline{a}
\end{aligned}
$$

$$
\begin{aligned}
\overrightarrow{A E} & =\overrightarrow{A B}+\overrightarrow{B C}+\overrightarrow{C D}+\overrightarrow{D E} \\
& =\underline{a}+\underline{b}-\underline{a}+\underline{b}-\underline{a} \\
& =2 \underline{b}-\underline{a}
\end{aligned}
$$

Any continuous route around the diagram from $A$ to $E$ will give the same answer

$$
\text { Try } \begin{aligned}
& A B O D E \\
& \overrightarrow{A E}=\overrightarrow{A B}+\overrightarrow{B O}+\overrightarrow{O D}+\overrightarrow{D E} \\
&=\underline{a}-\underline{a}+\underline{b}+\underline{b}-\underline{a} \\
&=2 \underline{b}-\underline{a}
\end{aligned}
$$

$3 J K L M N O P Q$ is a regular octagon.

$$
\overrightarrow{O J}=\mathbf{j} \quad \overrightarrow{O M}=\mathbf{m} \quad \overrightarrow{O P}=\mathbf{p}
$$


a) $\underset{\overrightarrow{N K}}{E_{\text {qua }}}$ to $j$
b) $E_{\text {qua }}$ to $m$ $\overrightarrow{Q K}$
c) $E \underset{L K}{ }$ val to f
d) $E_{\text {quart to }} \underset{\overrightarrow{T N}}{ }-j$
a j
b m
d $-\mathbf{j}$
c $p$
$f$ - $\mathbf{p}$
e) $\begin{gathered}E_{q u n} t_{0} \\ \vec{K} Q\end{gathered}$
f Equal to $\overrightarrow{k L}$
$1 W X Y Z$ is a square.
The diagonals $W Y$ and $X Z$ intersect at $M$.
$\overrightarrow{W X}=\mathbf{w}$ and $\overrightarrow{W Z}=\mathbf{z}$. Write these vectors in terms of $\mathbf{w}$ and $\mathbf{z}$.
a $\overrightarrow{X Y}$
b $\overrightarrow{W y}$

c $\overrightarrow{W M}$
f $\overrightarrow{X M}$
a) $\overrightarrow{x y}=z$
b) $\overrightarrow{w y}=\underline{w}+\underline{z}$
c) $\overrightarrow{W M}=\frac{1}{2} \overrightarrow{W M}=\frac{1}{2}(\underline{w}+\underline{z})$
d) $\overrightarrow{M y}=\frac{1}{2} \overrightarrow{\omega H}=\frac{1}{2}(\underline{w}+z)$
e) $\overrightarrow{z x}=-\underline{z}+\underline{\omega}, \overrightarrow{M x}=\frac{1}{2} \overrightarrow{z x}=\frac{1}{2}(-\underline{z}+\underline{w})$
f) $\overrightarrow{x M}=-\overrightarrow{M x}=-\frac{1}{2}(-\underline{2}+\underline{v})$ or $\frac{1}{2}(\underline{z}-\underline{w})$


Find $\overrightarrow{A B}$

$$
\begin{aligned}
\overrightarrow{A B} & =\overrightarrow{A O}+\overrightarrow{O B} \\
& =-\underline{a}+\underline{b}
\end{aligned}
$$

$P$ divides $A \Omega$ in the ratio $1: 4$

$$
\overrightarrow{A P}: \overrightarrow{P B}=1: 4
$$

Find $\overrightarrow{O P}$

$$
\begin{aligned}
\overrightarrow{O P} & =\overrightarrow{O A}+\overrightarrow{A P} \\
& =\overrightarrow{O A}+\frac{1}{5} \overrightarrow{A B}
\end{aligned}
$$

$$
\begin{aligned}
& =\underline{a}+\frac{1}{5}(-\underline{a}+b) \\
& =\underline{a}-\frac{1}{5} \underline{a}+\frac{1}{5} \underline{b} \\
& =\frac{4}{5} \underline{a}+\frac{1}{5} \underline{b}
\end{aligned}
$$

Similas Example

$Q$ divides $A B$ in the ratio 4:3

$$
\overrightarrow{A Q}: \overrightarrow{Q B}=4: 3
$$

i) Find $\overrightarrow{A B}$

$$
\begin{array}{rlr}
\overrightarrow{A B} & =\overrightarrow{A 0}+\overrightarrow{O B} \\
& =-\underline{a}+\underline{b} & 4+3=7 \text { parks }
\end{array}
$$

ii) Find $\overrightarrow{O Q}$

$$
\begin{aligned}
\overrightarrow{O Q} & =\overrightarrow{O A}+\overrightarrow{A Q} \\
& =\overrightarrow{O A}+\frac{4}{7} \overrightarrow{A B} \\
& =a+\frac{4}{7}(-\underline{a}+\underline{b}) \\
& =\underline{a}-\frac{4}{7} \underline{a}+\frac{4}{2} \underline{b} \\
\overrightarrow{O Q} & =\frac{3}{7} \underline{a}+\frac{4}{7} \underline{b}
\end{aligned}
$$

