

Quadratic Equations

Consider $(x+a)(x+b)$

$$= x^2 + ax + bx + ab$$

$$= x^2 + (a+b)x + ab$$

Factorising is the opposite of expanding

$$\begin{array}{rcl} x^2 + 5x + 6 & & \begin{array}{cc} +1 & +6 \\ -1 & -6 \\ +2 & +3 \checkmark \\ -2 & -3 \end{array} \\ = (x+2)(x+3) & & \end{array}$$

Ex1 Solve $x^2 + 5x + 6 = 0$

$$(x+2)(x+3) = 0$$

Either $x+2=0$ or $x+3=0$

$$\underline{x = -2}$$

$$\underline{x = -3}$$

Ex2

$$x^2 - 7x + 10 = 0$$

$$(x-2)(x-5) = 0$$

+1	+10
-1	-10
+2	+5
-2	-5 \checkmark

Either $x-2=0$ or $x-5=0$

$$\underline{x = 2}$$

$$\underline{x = 5}$$

E x 3

$$x^2 + x - 12 = 0$$

$$(x - 3)(x + 4) = 0$$

$$\begin{array}{r} +1 \quad -12 \\ -1 \quad +12 \\ +2 \quad -6 \\ -2 \quad +6 \\ +3 \quad -4 \\ -3 \quad +4 \end{array}$$

✓

Either $x - 3 = 0$ or $x + 4 = 0$

$x = 3$	$x = -4$
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E x 4

$$x^2 - 5x - 24 = 0$$

$$(x + 3)(x - 8) = 0$$

$$\begin{array}{r} +1 \quad -24 \\ -1 \quad +24 \\ +2 \quad -12 \\ -2 \quad +12 \\ +3 \quad -8 \end{array}$$

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Either $x + 3 = 0$ or $x - 8 = 0$

$x = -3$	$x = 8$
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Exercise

Solve

(Grade 5)

1) $x^2 - 5x - 14 = 0$
 $(x + 2)(x - 7) = 0$

Either $x + 2 = 0$ or $x - 7 = 0$

$$\underline{x = -2}$$

2) $x^2 + 11x + 10 = 0$
 $(x + 1)(x + 10) = 0$

Either $x + 1 = 0$ or $x + 10 = 0$

$$\underline{x = -1}$$

$$\underline{x = -10}$$

3) $x^2 + 4x - 21 = 0$
 $(x - 3)(x + 7) = 0$

Either $x - 3 = 0$ or $x + 7 = 0$

$$\underline{x = 3}$$

4) $x^2 - 2x + 1 = 0$
 $(x - 1)(x - 1) = 0$

Either $x - 1 = 0$ or $x - 1 = 0$

$$\underline{x = 1}$$

$$\underline{x = 1}$$

Called a double root

Multiple x^2 term

(Grade 7)

Ex 1

$$2x^2 + 9x + 10 = 0$$

$$\begin{array}{r} 2 \times 10 \\ = 20 \end{array}$$

$$+1 + 20$$

$$+2 + 10$$

$$+4 + 5 \checkmark$$

$$2x^2 + 4x + 5x + 10 = 0$$

$$2x(x+2) + 5(x+2) = 0$$

$$(2x+5)(x+2) = 0$$

Ex 1, the

$$2x+5=0$$

$$\text{or } x+2=0$$

$$2x = -5$$

$$x = -2$$

$$\underline{x = -\frac{5}{2}}$$

Ex 2

$$3x^2 + 2x - 8 = 0$$

$$\begin{array}{r} 3x-8 \\ = -24 \end{array}$$

$$\begin{array}{r} +1 -24 \\ -1 +24 \end{array}$$

$$+4 -6$$

$$-4 +6 \checkmark$$

$$3x^2 - 4x + 6x - 8 = 0$$

$$x(3x-4) + 2(3x-4) = 0$$

$$(x+2)(3x-4) = 0$$

$$\text{Ex 2, the } x+2=0 \quad \text{or } 3x-4=0$$

$$\underline{x = -2}$$

$$3x = 4$$

$$\underline{x = \frac{4}{3}}$$

Exercise

Solve

$$1) \quad 5x^2 + 8x + 3 = 0$$

$$\begin{array}{r} 5x^2 + 3x + 5x + 3 = 0 \\ \hline 2x(5x+3) + 1(5x+3) = 0 \\ (2x+1)(5x+3) = 0 \end{array}$$

Either $x+1=0$ or $5x+3=0$

$x = -1$ <hr/>	$5x = -3$ $x = -\frac{3}{5}$ <hr/>
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$$2) \quad 4x^2 - 8x - 5 = 0$$

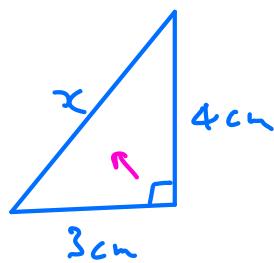
$$\begin{array}{r} 4x^2 + 2x - 10x - 5 = 0 \\ \hline 2x(2x+1) - 5(2x+1) = 0 \\ (2x-5)(2x+1) = 0 \end{array}$$

Either $2x-5=0$ or $2x+1=0$

$2x = 5$ $x = \frac{5}{2}$ <hr/>	$2x = -1$ $x = -\frac{1}{2}$ <hr/>
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Pythagoras Without a Calculator

Ex 1

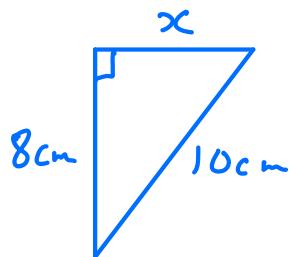


Find x

By Pythagorus

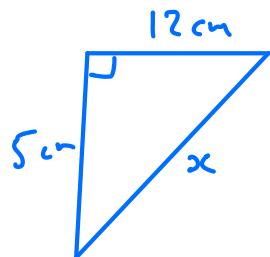
$$\begin{aligned} 3^2 + 4^2 &= x^2 \\ 9 + 16 &= x^2 \\ 25 &= x^2 \\ \sqrt{25} &= x \\ x &= 5 \text{ cm} \end{aligned}$$

Ex 2



$$\begin{aligned} x^2 + 8^2 &= 10^2 \\ x^2 &= 10^2 - 8^2 \\ x^2 &= 100 - 64 \\ x^2 &= 36 \\ x &= \sqrt{36} \\ x &= 6 \text{ cm} \end{aligned}$$

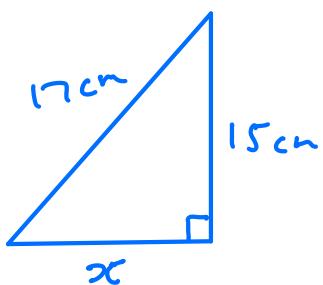
Ex 3



$$\begin{aligned} 5^2 + 12^2 &= x^2 \\ 25 + 144 &= x^2 \\ 169 &= x^2 \\ \sqrt{169} &= x \end{aligned}$$

$$x = 13 \text{ cm}$$

Ex 4



$$\begin{array}{r} 17 \\ 17 \\ \hline 119 \\ 170 \\ \hline 289 \end{array}$$

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 150 \\ \hline 225 \end{array}$$

$$\begin{aligned} x^2 + 15^2 &= 17^2 \\ x^2 &= 17^2 - 15^2 \\ x^2 &= 289 - 225 \\ x^2 &= 64 \\ x &= \sqrt{64} \\ x &= 8 \text{ cm} \end{aligned}$$

Pythagorean Triples

3, 4, 5

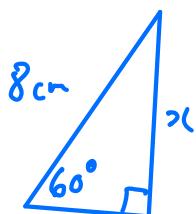
5, 12, 13

7, 24, 25

8, 15, 17

Trig Exact Values

	30°	45°	60°
\sin	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
\cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
\tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$



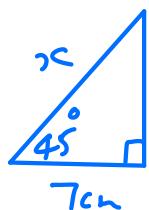
Find x

$$\sin 60^\circ = \frac{x}{8}$$

$$8 \sin 60^\circ = x$$

$$8 \times \frac{\sqrt{3}}{2} = x$$

$$x = 4\sqrt{3} \text{ cm}$$



$$\cos 45^\circ = \frac{7}{x}$$

$$\frac{1}{\sqrt{2}} = \frac{7}{x}$$

$$x = 7\sqrt{2}$$