Leave blank

2. Solve the equation

$$z^3 = 4\sqrt{2} - 4\sqrt{2}\mathrm{i},$$

giving your answers in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi < \theta \leqslant \pi$.

(6)

Leave blank

4. $z = -8 + (8\sqrt{3})\mathrm{i}$

(a) Find the modulus of z and the argument of z.

(3)

Using de Moivre's theorem,

(b) find z^3,

(2)

(c) find the values of w such that $w^4 = z$, giving your answers in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(5)

Leave blank

3. (a) Express the complex number $-2+\left(2\sqrt{3}\right)\mathrm{i}$ in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, $-\pi<\theta\leqslant\pi$.

(3)

(b) Solve the equation

$$z^4=-2+\left(2\sqrt{3}\right)\mathrm{i}$$

giving the roots in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, $-\pi<\theta\leqslant\pi$.

(5)

Leave blank

2. $z = 5\sqrt{3} - 5i$

Find

(a) $|z|$,

(1)

(b) $\arg(z)$, in terms of π.

(2)

$$w = 2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

Find

(c) $\left|\frac{w}{z}\right|$,

(1)

(d) $\arg\left(\frac{w}{z}\right)$, in terms of π.

(2)

Leave blank

4. (a) Given that

$$z = r(\cos\theta + \mathrm{i}\sin\theta), \qquad r \in \mathbb{R}$$

prove, by induction, that $z^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta), \qquad n \in \mathbb{Z}^+$

(5)

$$w = 3\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$$

(b) Find the exact value of w^5, giving your answer in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$.

(2)

Leave blank

Question 4 continued