Paper 1: Core Pure Mathematics

Topics	What students need to learn:		
	Content		Guidance
1 **Proof**	1.1	Construct proofs using mathematical induction. Contexts include sums of series, divisibility and powers of matrices.	To include induction proofs for: (i) summation of series, e.g. show $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$ or show $\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$ (ii) divisibility, e.g. show $3^{2n} + 11$ is divisible by 4 (iii) matrix products, e.g. show $\begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}^n = \begin{pmatrix} 2n+1 & -4n \\ n & 1-2n \end{pmatrix}$
2 **Complex numbers**	2.1	Solve any quadratic equation with real coefficients. Solve cubic or quartic equations with real coefficients.	Given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics, for example: (i) $\mathrm{f}(z) = 2z^3 + 5z^2 + 7z + 10$ Given that $z + 2$ is a factor of $\mathrm{f}(z)$, use algebra to solve $\mathrm{f}(z) = 0$ completely. (ii) $\mathrm{g}(x) = x^4 - x^3 + 6x^2 + 14x - 20$ Given $\mathrm{g}(1) = 0$ and $\mathrm{g}(-2) = 0$, use algebra to solve $\mathrm{g}(x) = 0$ completely.
	2.2	Add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$ with x and y real. Understand and use the terms 'real part' and 'imaginary part'.	Students should know the meaning of the terms, 'modulus' and 'argument'.
	2.3	Understand and use the complex conjugate. Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.	Knowledge that if z_1 is a root of $\mathrm{f}(z) = 0$ then $z_1{}^*$ is also a root.

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2 **Complex numbers** *continued*	2.4	Use and interpret Argand diagrams.	Students should be able to represent the sum or difference of two complex numbers on an Argand diagram.
	2.5	Convert between the Cartesian form and the modulus-argument form of a complex number. Knowledge of radians is assumed.	
	2.6	Multiply and divide complex numbers in modulus argument form. Knowledge of radians and compound angle formulae is assumed.	Knowledge of the results, $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \, \lvert z_2 \rvert$, $\left\lvert \frac{z_1}{z_2} \right\rvert = \frac{\lvert z_1 \rvert}{\lvert z_2 \rvert}$ $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$
	2.7	Construct and interpret simple loci in the argand diagram such as $\lvert z - a \rvert > r$ and $\arg(z - a) = \theta$ Knowledge of radians is assumed.	To include loci such as $\lvert z - a \rvert = b$, $\lvert z - a \rvert = \lvert z - b \rvert$, $\arg(z - a) = \beta$, and regions such as $\lvert z - a \rvert \leqslant \lvert z - b \rvert$, $\lvert z - a \rvert \leqslant b$, $\alpha < \arg(z - a) < \beta$
3 **Matrices**	3.1	Add, subtract and multiply conformable matrices. Multiply a matrix by a scalar.	
	3.2	Understand and use zero and identity matrices.	
	3.3	Use matrices to represent linear transformations in 2-D. Successive transformations. Single transformations in 3-D.	For 2-D, identification and use of the matrix representation of single and combined transformations from: reflection in coordinate axes and lines $y = \pm x$, rotation through any angle about $(0, 0)$, stretches parallel to the x-axis and y-axis, and enlargement about centre $(0, 0)$, with scale factor k, $(k \neq 0)$, where $k \in \mathbb{R}$. Knowledge that the transformation represented by $\mathbf{AB}$ is the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$. 3-D transformations confined to reflection in one of $x = 0$, $y = 0$, $z = 0$ or rotation about one of the coordinate axes. Knowledge of 3-D vectors is assumed.

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3 **Matrices** *continued*	3.4	Find invariant points and lines for a linear transformation.	For a given transformation, students should be able to find the coordinates of invariant points and the equations of invariant lines.
	3.5	Calculate determinants of: 2×2 and 3×3 matrices and interpret as scale factors, including the effect on orientation.	Idea of the determinant as an area scale factor in transformations.
	3.6	Understand and use singular and non-singular matrices. Properties of inverse matrices. Calculate and use the inverse of non-singular 2×2 matrices and 3×3 matrices.	Understanding the process of finding the inverse of a matrix is required. Students should be able to use a calculator to calculate the inverse of a matrix.
	3.7	Solve three linear simultaneous equations in three variables by use of the inverse matrix.	
	3.8	Interpret geometrically the solution and failure of solution of three simultaneous linear equations.	Students should be aware of the different possible geometrical configurations of three planes, including cases where the planes: (i) meet in a point (ii) form a sheaf (iii) form a prism or are otherwise inconsistent
4 **Further algebra and functions**	4.1	Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations.	For example, given a cubic polynomial equation with roots α, β and γ students should be able to evaluate expressions such as (i) $\alpha^2 + \beta^2 + \gamma^2$ (ii) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ (iii) $(3 + \alpha)(3 + \beta)(3 + \gamma)$ (iv) $\alpha^3 + \beta^3 + \gamma^3$

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4 **Further algebra and functions** *continued*	4.2	Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).	
	4.3	Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series.	For example, students should be able to sum series such as $\sum_{r+1}^{n} r(r^2+2)$
5 **Further calculus**	5.1	Derive formulae for and calculate volumes of revolution.	Both $\pi\int y^2 \ \mathrm{d}x$ and $\pi\int x^2 \ \mathrm{d}y$ are required.
6 **Further vectors**	6.1	Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.	The forms, $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and $\frac{x-a_1}{b_1} = \frac{x-a_2}{b_2} = \frac{x-a_3}{b_3}$ Find the point of intersection of two straight lines given in vector form. Students should be familiar with the concept of skew lines and parallel lines.
	6.2	Understand and use the vector and Cartesian forms of the equation of a plane.	The forms, $\mathbf{r}$ = a + λb + μc and ax + by + cz = d
	6.3	Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane.	$\mathbf{a.b} = \lvert\mathbf{a}\rvert \lvert\mathbf{b}\rvert \cos\theta$ The form $\mathbf{r.n} = k$ for a plane.
	6.4	Check whether vectors are perpendicular by using the scalar product.	Knowledge of the property that $\mathbf{a.b} = 0$ if the vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular.
	6.5	Find the intersection of a line and a plane. Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.	The perpendicular distance from (α, β, γ) to $n_1x + n_2y + n_3z + d = 0$ is $\frac{\lvert n_1\alpha + n_2\beta + n_3\gamma + d \rvert}{\sqrt{n_1^{\ 2} + n_2^{\ 2} + n_3^{\ 3}}}$

Assessment information

- First assessment: May/June 2018.
- The assessment is 1 hour 40 minutes.
- The assessment is out of 80 marks.
- Students must answer all questions.
- Calculators can be used in the assessment.
- The booklet 'Mathematical Formulae and Statistical Tables' will be provided for use in the assessment.

Sample assessment materials

A sample paper and mark scheme for this paper can be found in the *Pearson Edexcel Level 3 Advanced Subsidiary GCE in Further Mathematics Sample Assessment Materials (SAMs)* document.