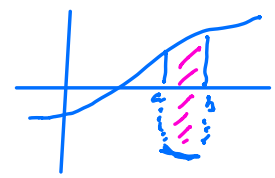
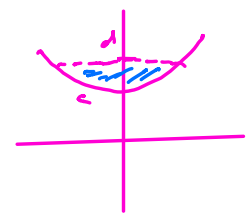
Volume of revolution about x-axis $= \pi \int_a^b y^2 \, dx$



Volume of revolution about y-axis $= \pi \int_d^c x^2 \, dy$



Exercise 4A

1 e) $y = \dfrac{\sqrt{\ln x}}{x}$ between $x = 1$ and $x = 2$

$$\text{Vol} = \pi \int_1^2 y^2 \, dx = \pi \int_1^2 \frac{\ln x}{x^2} \, dx$$

Let $u = \ln x$ $\Rightarrow \dfrac{du}{dx} = \dfrac{1}{x}$

Let $\dfrac{dv}{dx} = x^{-2}$ $\Rightarrow v = -x^{-1} = -\dfrac{1}{x}$

$$\pi \int_1^2 \frac{\ln x}{x^2} \, dx = uv - \int v \frac{du}{dx}$$

$$= \pi \left[\left[-\frac{\ln x}{x} \right]_1^2 - \int_1^2 -\frac{1}{x} \cdot \frac{1}{x} \, dx \right]$$

$$= \pi \left[\left[-\frac{\ln x}{x} \right]_1^2 + \int_1^2 \frac{1}{x^2} \, dx \right]$$

$$= \pi \left[-\frac{\ln 2}{2} - -\frac{\ln 1}{1} + \left[-\frac{1}{x} \right]_1^2 \right]$$

$$= \pi\left[-\frac{\ln 2}{2} - \frac{1}{2} - - \frac{1}{1}\right]$$

$$= \pi\left[\frac{1}{2} - \frac{\ln 2}{2}\right] = \frac{\pi}{2}(1 - \ln 2)$$

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1 f) $y = \operatorname{cosec} x + \cot x$ from $x = \frac{\pi}{3}$ to $x = \frac{\pi}{2}$

$$\text{Vol} = \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} y^2 \, dx = \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(\operatorname{cosec}^2 x + \cot^2 x + 2 \operatorname{cosec} x \cot x\right) dx$$

$$= \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left(2 \operatorname{cosec}^2 x - 1 + 2 \operatorname{cosec} x \cot x\right) dx$$

$$= \pi \left[-2 \cot x - x\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{2 \cos x}{\sin^2 x} \, dx$$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

$x = \frac{\pi}{3} \quad u = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{2} \quad u = 1$

$$\int_{\frac{\sqrt{3}}{2}}^{1} \frac{2}{u^2} \, du$$

$$\pi\left[\left(0 - \frac{\pi}{2}\right) - \left(-\frac{2}{\sqrt{3}} - \frac{\pi}{3}\right)\right]$$

$$+ \pi \left[-\frac{2}{u}\right]_{\frac{\sqrt{3}}{2}}^{1}$$

$$= \pi\left(-\frac{\pi}{6} + \frac{2}{\sqrt{3}}\right)$$

$$+ \pi\left(-2 + \frac{4}{\sqrt{3}}\right)$$

$$= \pi\left(\frac{6}{\sqrt{3}} - 2 - \frac{\pi}{6}\right) = \pi\left(2\sqrt{3} - 2 - \frac{\pi}{6}\right)$$