

8.2 Simple harmonic motion

You can use second – order differential equations to model particles moving with **simple harmonic motion**.

- **Simple harmonic motion (S.H.M.)** is motion in which the acceleration of a particle P is always towards a fixed point O on the line of motion of P . The acceleration is proportional to the displacement of P from O .

$$\dot{x} = \frac{dx}{dt} = v$$

$$x'' = -\omega^2 x$$

$$x'' = \frac{d^2x}{dt^2} = \frac{dv}{dt}$$

$$\begin{aligned} \text{Period of oscillation} \\ = \frac{2\pi}{\omega} \end{aligned}$$

$$x'' = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v \frac{dv}{dx}$$

Exercise 8B

i) $\frac{d^2x}{dt^2} = -9x$ a) SHM

b) $t = 0s, x = 2m, v = 3ms^{-1}$

$$\frac{d^2x}{dt^2} + 9x = 0$$

$$\begin{aligned} \text{Aux Eqn} \\ m^2 + 9 = 0 \\ m = \pm 3i \end{aligned}$$

$$x = A \cos 3t + B \sin 3t$$

$$2 = A \cos 0 + B \sin 0$$

$$2 = A$$

$$v = -3A \sin 3t + 3B \cos 3t$$

$$3 = -3A \sin 0 + 3B \cos 0$$

$$\begin{array}{l} 3 = 3B \\ 1 = B \end{array}$$

$$x = 2\cos 3t + \sin 3t$$

$$x = \sqrt{5} \cos(3t - \alpha)$$

$$\text{where } \alpha = \tan^{-1} \frac{1}{2}$$

$$\text{Max distance} = \sqrt{5}$$

3) a) SHM

b) $\ddot{x} = -5 \text{ ms}^{-2}$ when $x = 1$

$$\ddot{x} = -\omega^2 x$$

$$-5 = -\omega^2 \times 1 \Rightarrow \omega = \sqrt{5}$$

$$\therefore \ddot{x} = -5x$$

c) $t = 0s, v = 6 \text{ ms}^{-1}, x = 5 \text{ m}$

$$x = A \cos \sqrt{5}t + B \sin \sqrt{5}t$$

$t = 0, 5 = A$

$$v = -\sqrt{5}A \sin \sqrt{5}t + \sqrt{5}B \cos \sqrt{5}t$$

$t = 0, 6 = \sqrt{5}B$

$$\frac{6}{\sqrt{5}} = B$$

$$x = 5 \cos \sqrt{5}t + \frac{6\sqrt{5}}{5} \sin \sqrt{5}t$$

d) Max Displacement = $\sqrt{5^2 + \left(\frac{6\sqrt{5}}{5}\right)^2}$

$$= \sqrt{25 + \frac{36}{5}}$$

$$= \frac{\sqrt{805}}{5}$$
