n^{th} roots of unity

$$\cos\frac{2\pi k}{n} + i\sin\frac{2\pi k}{n} \quad \text{for } k = 0, 1, 2, \ldots n-1$$

Example 3rd roots (cubic roots) of unity

$$\cos 0 + i\sin 0 = 1$$

$$\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$$

$$\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

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n^{th} roots of any complex number W

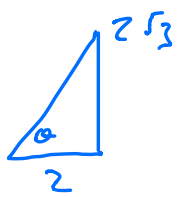
$$W = r(\cos\phi + i\sin\phi)$$

n^{th} roots are $r^{\frac{1}{n}}\left(\cos\left(\frac{\phi + 2k\pi}{n}\right) + i\sin\left(\frac{\phi + 2k\pi}{n}\right)\right)$

for $k = 0, 1, 2, \ldots n-1$

Ex 16 $\quad z^4 = 2 + i2\sqrt{3}$

$$|z^4| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = \sqrt{16} = 4$$



$$\theta = \tan^{-1}\frac{2\sqrt{3}}{2} = \frac{\pi}{3}$$

$$z = 4^{\frac{1}{4}}\left(\cos\left(\frac{\frac{\pi}{3} + 2k\pi}{4}\right) + i\sin\left(\frac{\frac{\pi}{3} + 2k\pi}{4}\right)\right)$$

for $k = 0, 1, 2, 3$

$|z| = \sqrt{2}$ $\quad \theta = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}, \frac{19\pi}{12}$

$\theta = \frac{\pi}{12}, \frac{7\pi}{12}, -\frac{11\pi}{12}, -\frac{5\pi}{12}$

$$z = \sqrt{2}e^{i\frac{\pi}{12}} \quad \sqrt{2}e^{i\frac{7\pi}{12}} \quad \sqrt{2}e^{-i\frac{11\pi}{12}} \quad \sqrt{2}e^{-i\frac{5\pi}{12}}$$

Exercise 1F

1 a) $z^4 - 1 = 0 \Rightarrow z^4 = 1 \Rightarrow z = 1, -1, i, -i$

1 b) $z^3 - i = 0 \quad z^3 = +i = 1\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

$$z = \cos\left(\frac{\frac{\pi}{2} + 2k\pi}{3}\right) + i\sin\left(\frac{\frac{\pi}{2} + 2k\pi}{3}\right)$$

$$z = e^{i\frac{\pi}{6}}, \quad e^{i\frac{5\pi}{6}}, \quad e^{-i\frac{\pi}{2}}$$

$$z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$z = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$$

$$z = -i$$

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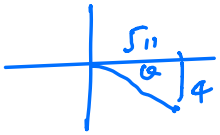
3 b) $z^3 = \sqrt{11} - 4i$

$|z|^3 = \sqrt{11 + 16} = \sqrt{27} = 27^{\frac{1}{2}}$

$|z| = \left(27^{\frac{1}{2}}\right)^{\frac{1}{3}} = 27^{\frac{1}{6}} = \sqrt{3}$

$\arg z^3 = \theta = -\tan^{-1}\frac{4}{\sqrt{11}} = -0.8785$



$$z = \sqrt{3}\left(\cos\left(\frac{-0.8785 + 2k\pi}{3}\right) + i\sin\left(\frac{-0.8785 + 2\pi k}{3}\right)\right)$$

$$z = \sqrt{3}e^{-0.29i}, \quad \sqrt{3}e^{1.80i}, \quad \sqrt{3}e^{-2.39i}$$

Hwk 2 parts from each of

Exercise 1F Q 1, 2, 3