

Exercise 5C

$$6) \quad r = 3 \sin \theta \quad \textcircled{1}$$

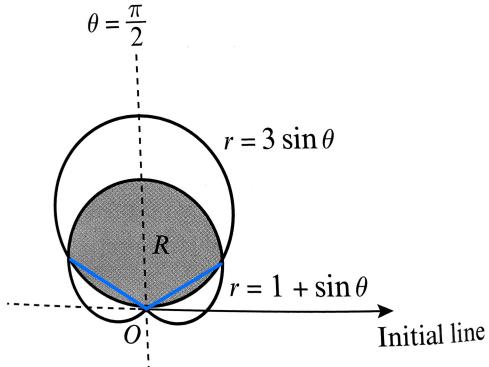
$$r = 1 + \sin \theta \quad \textcircled{2}$$

$$\textcircled{1} - \textcircled{2} \quad 0 = 2 \sin \theta - 1$$

$$1 = 2 \sin \theta$$

$$\frac{1}{2} = \sin \theta$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$



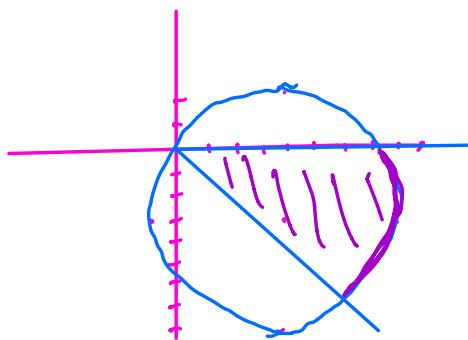
Both curves symmetrical about $\theta = \frac{\pi}{2}$

$$\begin{aligned} \therefore R &= 2 \left[\frac{1}{2} \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1 + \sin \theta)^2 d\theta + \frac{1}{2} \int_0^{\frac{\pi}{6}} (3 \sin \theta)^2 d\theta \right] \\ &= \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} (1 + 2 \sin \theta + \sin^2 \theta) d\theta + \int_0^{\frac{\pi}{2}} 9 \sin^2 \theta d\theta \\ &= \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \left(1 + 2 \sin \theta + \frac{1 - \cos 2\theta}{2} \right) d\theta + \int_0^{\frac{\pi}{2}} \frac{9}{2} (1 - \cos 2\theta) d\theta \\ &= \left[\frac{3\theta}{2} - 2 \cos \theta - \frac{\sin 2\theta}{4} \right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} + \left[\frac{9}{2} \left(\theta - \frac{\sin 2\theta}{2} \right) \right]_0^{\frac{\pi}{2}} \\ &= \left(\frac{3\pi}{4} - 0 + 0 \right) - \left(\frac{\pi}{4} - \sqrt{3} - \frac{\sqrt{3}}{8} \right) + \left(\frac{3\pi}{4} - \frac{9\sqrt{3}}{8} \right) - (0 - 0) \\ &= \frac{3\pi}{4} - \frac{\pi}{4} + \sqrt{3} + \frac{\sqrt{3}}{8} + \frac{3\pi}{4} - \frac{9\sqrt{3}}{8} \end{aligned}$$

$$= \frac{5\pi}{4}$$

7)

a)



$$z = (4 - 3i)$$

$$r = 8\cos\theta - 6\sin\theta$$

$$\text{Area} = \frac{1}{2} \int_{-\frac{\pi}{4}}^0 r^2 d\theta$$

$$= \frac{1}{2} \int_{-\frac{\pi}{4}}^0 (10\cos(\theta + \alpha))^2 d\theta$$

$$\alpha = \tan^{-1} \frac{6}{8}$$

$$\alpha = 0.6435 \text{ rad} = 50 \int_{-\frac{\pi}{4}}^0 \frac{1 + \cos(2\theta + 2\alpha)}{2} d\theta$$

$$= 50 \left[\frac{\theta}{2} + \frac{\sin(2\theta + 2\alpha)}{4} \right]_{-\frac{\pi}{4}}^0$$

$$= 50 \left[0 + \frac{\sin 2\alpha}{4} - \left(-\frac{\pi}{8} + \frac{\sin(2\alpha - \frac{\pi}{2})}{4} \right) \right]$$

$$= 35.1$$
