Edexcel FP2

Polar Coordinates

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4.

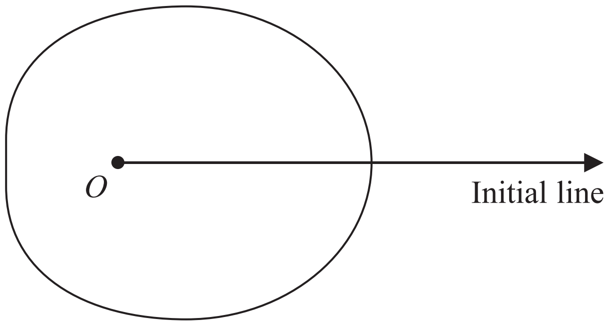

Figure 1

Figure 1 shows a sketch of the curve with polar equation

$$r = a + 3\cos\theta, \quad a > 0, \quad 0 \leqslant \theta < 2\pi$$

The area enclosed by the curve is $\frac{107}{2}\pi$.

Find the value of a.

(8)

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Question 4 continued

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5.

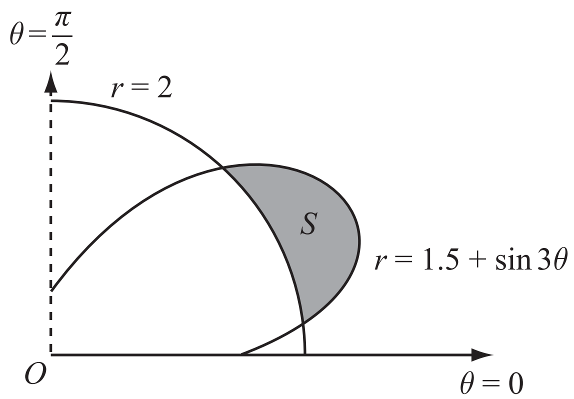

Figure 1

Figure 1 shows the curves given by the polar equations

$$r = 2, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2},$$

$$\text{and} \qquad r = 1.5 + \sin 3\theta, \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the coordinates of the points where the curves intersect.

(3)

The region S, between the curves, for which $r > 2$ and for which $r < (1.5 + \sin 3\theta)$, is shown shaded in Figure 1.

(b) Find, by integration, the area of the shaded region S, giving your answer in the form $a\pi + b\sqrt{3}$, where a and b are simplified fractions.

(7)

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Question 5 continued

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6.

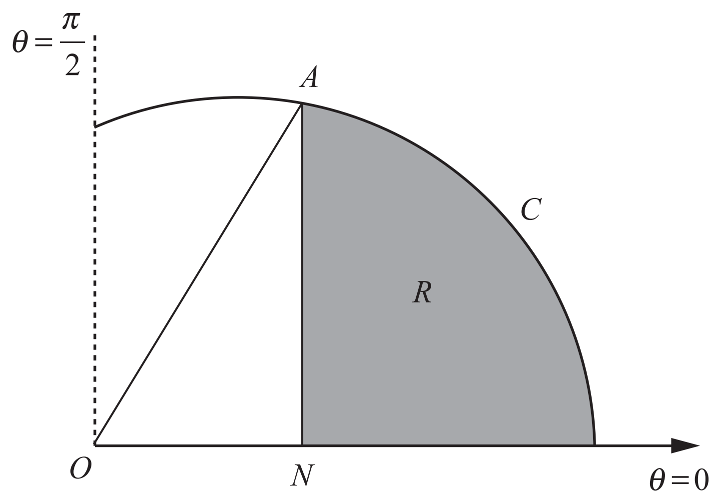

Figure 1

The curve C shown in Figure 1 has polar equation

$$r = 2 + \cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point A on C, the value of r is $\frac{5}{2}$.

The point N lies on the initial line and AN is perpendicular to the initial line.

The finite region R, shown shaded in Figure 1, is bounded by the curve C, the initial line and the line AN.

Find the exact area of the shaded region R.

(9)

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Question 6 continued

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2. The curve C has polar equation

$$r = 1 + 2\cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

At the point P on C, the tangent to C is parallel to the initial line.

Given that O is the pole, find the exact length of the line OP.

(7)

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8.

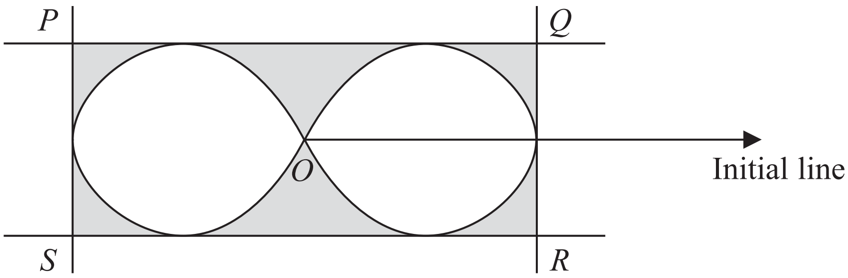

Figure 1

Figure 1 shows a closed curve C with equation

$$r = 3(\cos 2\theta)^{\frac{1}{2}}, \qquad \text{where } -\frac{\pi}{4} < \theta \leqslant \frac{\pi}{4},\ \frac{3\pi}{4} < \theta \leqslant \frac{5\pi}{4}$$

The lines PQ, SR, PS and QR are tangents to C, where PQ and SR are parallel to the initial line and PS and QR are perpendicular to the initial line. The point O is the pole.

(a) Find the total area enclosed by the curve C, shown unshaded inside the rectangle in Figure 1.

(4)

(b) Find the total area of the region bounded by the curve C and the four tangents, shown shaded in Figure 1.

(9)

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Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END

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8.

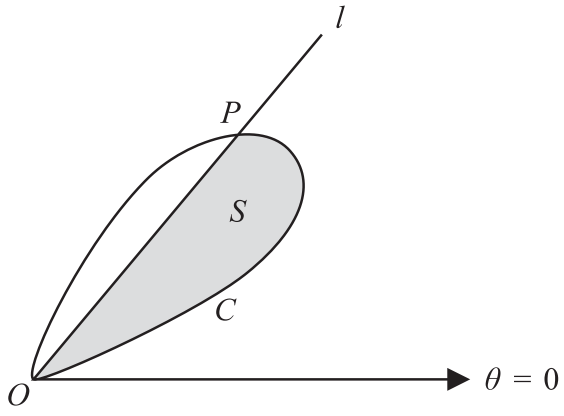

Figure 1

Figure 1 shows a curve C with polar equation $r = a\sin 2\theta,\ 0 \leqslant \theta \leqslant \frac{\pi}{2}$, and a half-line l.

The half-line l meets C at the pole O and at the point P. The tangent to C at P is parallel to the initial line. The polar coordinates of P are (R, ϕ).

(a) Show that $\cos\phi = \frac{1}{\sqrt{3}}$

(6)

(b) Find the exact value of R.

(2)

The region S, shown shaded in Figure 1, is bounded by C and l.

(c) Use calculus to show that the exact area of S is

$$\frac{1}{36}a^2\left(9\arccos\left(\frac{1}{\sqrt{3}}\right) + \sqrt{2}\right)$$

(7)

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Question 8 continued

Q8

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

END

P 4 3 1 4 9 A 0 2 8 2 8

Further Pure Mathematics FP2

Candidates sitting FP2 may also require those formulae listed under Further Pure Mathematics FP1 and Core Mathematics C1–C4.

Area of a sector

$$A = \frac{1}{2}\int r^2 \ \mathrm{d}\theta \qquad \text{(polar coordinates)}$$

Complex numbers

$$\mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$$

$$\{r(\cos\theta + \mathrm{i}\sin\theta)\}^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$$

The roots of $z^n = 1$ are given by $z = \mathrm{e}^{\frac{2\pi k\mathrm{i}}{n}}$, for $k = 0, 1, 2, \ldots, n-1$

Maclaurin's and Taylor's Series

$$\mathrm{f}(x) = \mathrm{f}(0) + x\,\mathrm{f}'(0) + \frac{x^2}{2!}\mathrm{f}''(0) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(0) + \ldots$$

$$\mathrm{f}(x) = \mathrm{f}(a) + (x-a)\,\mathrm{f}'(a) + \frac{(x-a)^2}{2!}\mathrm{f}''(a) + \ldots + \frac{(x-a)^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{f}(a+x) = \mathrm{f}(a) + x\,\mathrm{f}'(a) + \frac{x^2}{2!}\mathrm{f}''(a) + \ldots + \frac{x^r}{r!}\mathrm{f}^{(r)}(a) + \ldots$$

$$\mathrm{e}^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \ldots + \frac{x^r}{r!} + \ldots \qquad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{r+1}\frac{x^r}{r} + \ldots \qquad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \ldots \qquad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^r \frac{x^{2r}}{(2r)!} + \ldots \qquad \text{for all } x$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^r \frac{x^{2r+1}}{2r+1} + \ldots \qquad (-1 \leq x \leq 1)$$

Further Pure Mathematics FP1

Candidates sitting FP1 may also require those formulae listed under Core Mathematics C1 and C2.

Summations

$$\sum_{r=1}^{n} r^2 = \tfrac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \tfrac{1}{4}n^2(n+1)^2$$

Numerical solution of equations

The Newton-Raphson iteration for solving $\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Conics

	Parabola	**Rectangular Hyperbola**
Standard Form	$y^2 = 4ax$	$xy = c^2$
Parametric Form	$(at^2, 2at)$	$\left(ct, \dfrac{c}{t}\right)$
Foci	$(a, 0)$	Not required
Directrices	$x = -a$	Not required

Matrix transformations

Anticlockwise rotation through θ about O: $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Reflection in the line $y = (\tan\theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

In FP1, θ will be a multiple of 45°.

Core Mathematics C4

Candidates sitting C4 may also require those formulae listed under Core Mathematics C1, C2 and C3.

Integration **(+ *constant*)**

$\mathbf{f}(\boldsymbol{x})$	$\int \mathbf{f}(\boldsymbol{x}) \; \mathbf{d}\boldsymbol{x}$
$\sec^2 kx$	$\dfrac{1}{k} \tan kx$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\operatorname{cosec} x$	$-\ln\lvert\operatorname{cosec} x + \cot x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x)\rvert$
$\sec x$	$\ln\lvert\sec x + \tan x\rvert, \quad \ln\lvert\tan(\tfrac{1}{2}x + \tfrac{1}{4}\pi)\rvert$

$$\int u \frac{\mathrm{d}v}{\mathrm{d}x} \mathrm{d}x = uv - \int v \frac{\mathrm{d}u}{\mathrm{d}x} \mathrm{d}x$$

Core Mathematics C3

Candidates sitting C3 may also require those formulae listed under Core Mathematics C1 and C2.

Logarithms and exponentials

$$e^{x \ln a} = a^x$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad (A \pm B \neq (k + \tfrac{1}{2})\pi)$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Differentiation

$\mathbf{f}(\boldsymbol{x})$	$\mathbf{f}'(\boldsymbol{x})$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2\, x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$
$\dfrac{\mathrm{f}(x)}{\mathrm{g}(x)}$	$\dfrac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{(\mathrm{g}(x))^2}$

Core Mathematics C2

Candidates sitting C2 may also require those formulae listed under Core Mathematics C1.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Binomial series

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N})$$

$$\text{where } \binom{n}{r} = {}^n\mathrm{C}_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times\ldots\times r}x^r + \ldots \qquad (|x|<1, n \in \mathbb{R})$$

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Numerical integration

The trapezium rule: $\int_a^b y \, \mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

Core Mathematics C1

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Arithmetic series

$u_n = a + (n-1)d$

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n[2a+(n-1)d]$