

Vertical and Horizontal Lines

$$x = r \cos \theta$$

$$\frac{x}{\cos \theta} = r$$

$$r = x \sec \theta$$

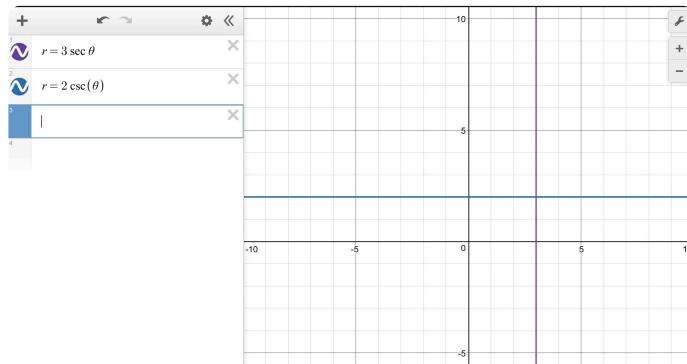
eg $r = 3 \sec \theta$

$$y = r \sin \theta$$

$$\frac{y}{\sin \theta} = r$$

$$r = y \csc \theta$$

eg $r = 2 \csc \theta$



Mixed Exercise 5

D) $r = a \left(1 + \frac{1}{2} \sin \theta \right)$ $0 \leq \theta < 2\pi$

$$\text{Area} = \frac{1}{2} \int_0^{2\pi} r^2 d\theta$$

$$= \frac{1}{2} a^2 \int_0^{2\pi} \left(1 + \sin \theta + \frac{1}{4} \sin^2 \theta \right) d\theta$$

$$= \frac{1}{2} a^2 \int_0^{2\pi} \left(1 + \sin \theta + \frac{1 - \cos 2\theta}{8} \right) d\theta$$

$$= \frac{1}{2} a^2 \left[\frac{9a}{8} - \cos\theta - \frac{\sin 2\theta}{16} \right]_0^{2\pi}$$

$$= \frac{1}{2} a^2 \left[\left(\frac{9\pi}{4} - 1 - 0 \right) - (0 - 1 - 0) \right]$$

$$= \frac{9a^2\pi}{8}$$

2) $r = a(1 + \cos\theta)$

a)

b)

c) $2a \sec\theta = 2a(1 + \cos\theta)$

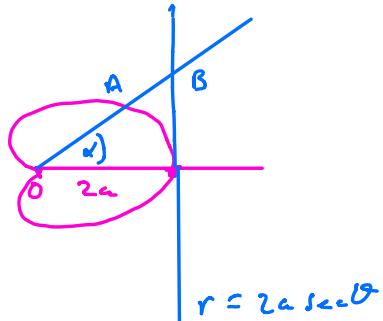
$$\frac{1}{\cos\theta} = 1 + \cos\theta$$

$$1 = \cos\theta + \cos^2\theta$$

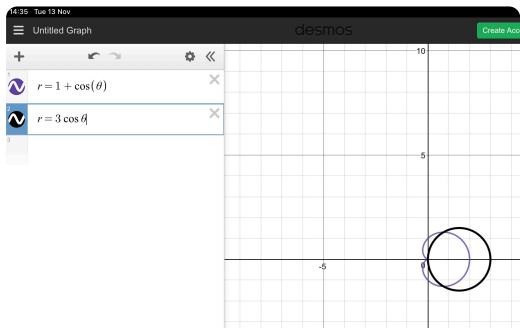
$$\cos^2\theta + \cos\theta - 1 = 0$$

$$\cos\theta = \frac{-1 \pm \sqrt{5}}{2}$$

$$\cos\theta = \frac{-1 + \sqrt{5}}{2}$$



3)



$$r = 1 + \cos\theta$$

$$r = 3 \cos\theta$$

Intersect when

$$1 + \cos\theta = 3 \cos\theta$$

$$1 = 2 \cos\theta$$

$$\frac{1}{2} = \cos\theta$$

$$\theta = \frac{\pi}{3}$$

Area inside curves

$$= 2 \int_0^{\frac{\pi}{3}} \frac{1}{2} r_1^2 + 2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2} r_2^2 d\theta$$

$$= 2 \cdot \frac{1}{2} \int_0^{\frac{\pi}{3}} (1 + \cos\theta)^2 d\theta + 2 \cdot \frac{1}{2} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 9 \cos^2\theta d\theta$$

$$= \int_0^{\frac{\pi}{3}} \left(1 + 2\cos\theta + \frac{1+\cos 2\theta}{2} \right) d\theta + 9 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2} d\theta$$

$$= \left[\frac{3\theta}{2} + 2\sin\theta + \frac{\sin 2\theta}{4} \right]_0^{\frac{\pi}{3}} + 9 \left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$= \left(\frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} \right) - (0 + 0 + 0) + 9 \left[\left(\frac{\pi}{4} + 0 \right) - \left(\frac{\pi}{6} + \frac{\sqrt{3}}{8} \right) \right]$$

$$= \frac{\pi}{2} + \sqrt{3} + \frac{\sqrt{3}}{8} + \frac{9\pi}{4} - \frac{9\pi}{6} - \frac{9\sqrt{3}}{8}$$

$$= \frac{5\pi}{4}$$

4) $r^2 = a^2 \sin 2\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$

$x = r \cos \theta$

$x = a (\sin 2\theta)^{\frac{1}{2}} \cos \theta$

$\frac{dx}{d\theta} = a (\sin 2\theta)^{\frac{1}{2}} (-\sin \theta) + \cos \theta a \frac{1}{2} (\sin 2\theta)^{-\frac{1}{2}} 2 \cos 2\theta$

$= -a \sin \theta \sqrt{\sin 2\theta} + \frac{a \cos \theta \cos 2\theta}{\sqrt{\sin 2\theta}}$

Is it \perp to initial line when $\frac{dx}{d\theta} = 0$

$$\Rightarrow -\sin \theta \sqrt{\sin 2\theta} + \frac{\cos \theta \cos 2\theta}{\sqrt{\sin 2\theta}} = 0$$

$$-\sin \theta \sin 2\theta + \cos \theta \cos 2\theta = 0$$

$$\cos(2\theta + \theta) = 0$$

$$\cos 3\theta = 0$$

$$\Rightarrow 3\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2},$$

$$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}$$

$$\theta = \frac{\pi}{6} \quad r^2 = a^2 \sin \frac{\pi}{3} = \frac{a^2 \sqrt{3}}{2} \quad r = a \sqrt{\frac{\sqrt{3}}{2}}$$

$$\theta = \frac{\pi}{2} \quad r^2 = a^2 \sin \pi = 0 \quad r = 0$$

$$\theta = \frac{5\pi}{6} \quad r^2 = a^2 \sin \frac{10\pi}{6} = -a^2 \frac{\sqrt{3}}{2} \quad \text{no root}$$

$$\theta = \frac{7\pi}{6} \quad r^2 = a^2 \sin \frac{14\pi}{6} = a^2 \frac{\sqrt{3}}{2} \quad r = a \sqrt{\frac{\sqrt{3}}{2}}$$

Solution $\left(a\sqrt{\frac{\sqrt{3}}{2}}, \frac{\pi}{6} \right) \quad \left(0, \frac{\pi}{2} \right) \quad \left(a\sqrt{\frac{\sqrt{3}}{2}}, \frac{7\pi}{6} \right)$
