

$$45 \quad \int_{-1}^3 \frac{x-1}{\sqrt{3+2x-x^2}} dx = \int_1^3 \frac{x-1}{\sqrt{(3-x)(1+x)}} dx$$

$$\stackrel{\lim_{t \rightarrow 0}}{=} \int_0^0 \frac{x-1}{\sqrt{3+2x-t^2}} dx + (*)$$

$$\stackrel{\lim_{t \rightarrow 1^-}}{=} \int_{3+2t-t^2}^3 -\frac{1}{2\sqrt{u}} du$$

$$= \left[ -\sqrt{u} \right]_{3+2t-t^2}^3$$

$$= -\sqrt{3} - -\sqrt{3+2t-t^2}$$

$$\stackrel{\lim_{t \rightarrow 1^-}}{=} \left[ -\sqrt{3} + \sqrt{3+2t-t^2} \right] = \left[ -\sqrt{3} + \sqrt{3-2+1} \right] = -\sqrt{3}$$

$$(*) \stackrel{\lim_{t \rightarrow 3}}{=} \int_0^3 \frac{x-1}{\sqrt{3+2x-x^2}} dx = \int_3^{3+2t-t^2} -\frac{1}{2\sqrt{u}} du$$

$$\stackrel{\lim_{t \rightarrow 3}}{=} \left[ -\sqrt{u} \right]_3^{3+2t-t^2}$$

$$= -\sqrt{3+6-9} - -\sqrt{3}$$

$$= +\sqrt{3}$$

$\therefore$  Integral converges

$$\text{Value} = -\sqrt{3} + \sqrt{3} = 0$$

$$\text{Let } u = 3+2x-x^2$$

$$\frac{du}{dx} = 2-2x$$

$$du = (2-2x)dx$$

$$-\frac{1}{2}du = (x-1)dx$$

$$x=0 \quad u=3$$

$$x=t \quad u=3+2t-t^2$$

Exercise 3A Q8

$$\int (\ln x)^2 dx$$

$$\text{Let } u = (\ln x)^2 \quad \frac{du}{dx} = 1 \\ \frac{du}{dx} = \frac{2 \ln x}{x} \quad v = x$$

$$\begin{aligned} \int u \frac{dv}{dx} dx &= uv - \int v \frac{du}{dx} dx \\ &= x(\ln x)^2 - \int \cancel{x \cdot 2 \ln x} dx \\ &= x(\ln x)^2 - 2(x \ln x - x) \\ &= x(\ln x)^2 - 2x \ln x + 2x + C \end{aligned}$$


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$$\int_0^1 (\ln x)^2 dx = \lim_{\epsilon \rightarrow 0} \int_\epsilon^1 (\ln x)^2 dx$$

$$\lim_{\epsilon \rightarrow 0} \left[ x(\ln x)^2 - 2x \ln x + 2x \right]_0^1$$

$$\lim_{\epsilon \rightarrow 0} \left[ x((\ln x)^2 - 2 \ln x + 2) \right]_0^1$$

$$\rightarrow (1(0 - 0 + 2) - (0 - 0 + 2))$$

convergent

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Q10

$$\int_{-2}^2 \frac{\sqrt{2-x} - 3\sqrt{2+x}}{\sqrt{4-x^2}} dx$$

Consider  $\int_\epsilon^0 + \int_0^\epsilon$

$$\int_0^\epsilon \frac{\sqrt{2-x}}{\sqrt{4-x^2}} dx + \int_0^\epsilon -\frac{3\sqrt{2+x}}{\sqrt{4-x^2}} dx$$

$$\int_0^t \frac{\sqrt{2-x}}{\sqrt{2-x} \sqrt{2+x}} dx + \int_0^t \frac{-3\sqrt{2+x}}{\sqrt{2-x} \sqrt{2+x}} dx$$

$$= \lim_{k \rightarrow 0} \int_0^t \frac{1}{\sqrt{2+kx}} dx - 3 \int_0^t \frac{1}{\sqrt{2-x}} dx$$



$$\text{Let } u = 2+x$$

$$\frac{du}{dx} = 1$$

$$du = dx$$

$$\int_2^{t+2} \frac{1}{\sqrt{u}} du$$

$$= \left[ 2\sqrt{u} \right]_2^{t+2}$$

$$\text{Let } u = 2-x$$

$$\frac{du}{dx} = -1$$

$$du = -dx$$

$$-3 \int_2^{2-t} -\frac{1}{\sqrt{u}} du$$

$$= -3 \left[ -2\sqrt{u} \right]_2^{2-t}$$

$$\text{As } t \rightarrow 2 \rightarrow 2\sqrt{4} - 2\sqrt{2} + 6\sqrt{0} - 6\sqrt{2}$$

$$= 4 - 8\sqrt{2}$$


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$$\int_0^0 = \left[ 2\sqrt{u} \right]_{2+t}^2 - 3 \left[ -2\sqrt{u} \right]_{2-t}^2$$

$$t \rightarrow -2 \quad 2\sqrt{2} - 2\sqrt{0} - 3(-2\sqrt{2} + 2\sqrt{4})$$

$$2\sqrt{2} + 6\sqrt{2}$$

$$= 8\sqrt{2} - 12$$

$$\text{Overall integral} = 4 - 8\sqrt{2} + 8\sqrt{2} - 12$$

$$= -8$$