

Exercise 3A Q7

$$\int \frac{\ln x}{x} dx$$

$$= \int u du$$

$$= \frac{u^2}{2} + C$$

$$= \frac{(\ln x)^2}{2} + C$$

$$\text{Let } u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$du = \frac{1}{x} dx$$

$$\int \frac{\ln x}{x} dx$$

Integration by Parts

$$\text{Let } u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\text{Let } \frac{dv}{dx} = \frac{1}{x}$$

$$v = \ln x$$

$$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\int \frac{\ln x}{x} dx = (\ln x)^2 - \int \frac{\ln x}{x} dx$$

$$2 \int \frac{\ln x}{x} dx = (\ln x)^2 + C$$

$$\int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} + C$$

$$7b) \quad \int_1^\infty \frac{\ln x}{x} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{\ln x}{x} dx$$

$$= \lim_{t \rightarrow \infty} \left[\frac{(\ln x)^2}{2} \right]_1^t$$

undefined as $\log t \rightarrow \infty$
 as $t \rightarrow \infty$

$$\begin{aligned}
 \text{(a)} \quad & \int x^2 e^{x^3} dx \\
 &= \int \frac{1}{3} e^u du \quad \text{Let } u = x^3 \\
 &= \frac{1}{3} e^u + C \quad \frac{du}{dx} = 3x^2 \\
 &= \frac{1}{3} e^{x^3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad & \int_{-\infty}^1 x^2 e^{x^3} dx = \lim_{\epsilon \rightarrow -\infty} \int_{\epsilon}^1 x^2 e^{x^3} dx \\
 &= \lim_{\epsilon \rightarrow -\infty} \left[\frac{1}{3} e^{x^3} \right]_{\epsilon}^1 \\
 &= \lim_{\epsilon \rightarrow -\infty} \left[\frac{1}{3} e^1 - \frac{1}{3} e^{\epsilon} \right] \\
 &\quad e^{\epsilon} \rightarrow 0 \text{ as } \epsilon \rightarrow -\infty \\
 &\therefore \lim_{\epsilon \rightarrow -\infty} \left[\frac{1}{3} e^1 - \frac{1}{3} e^{\epsilon} \right] \\
 &= \frac{1}{3} e^1 - 0 \\
 &= \frac{1}{3} e \quad \therefore \text{converges}
 \end{aligned}$$

$$\text{(15a)} \quad \int_0^K \frac{1}{2x^2 + 3x + 1} dx = \int_0^K \frac{1}{(2x+1)(x+1)} dx$$

$$\begin{aligned}
 &= \int_0^k \left(\frac{2}{2x+1} - \frac{1}{x+1} \right) dx \\
 &= \ln(2x+1) - \ln(x+1) + C \\
 &= \ln\left(\frac{2k+1}{k+1}\right) + C
 \end{aligned}$$

155)

$$\begin{aligned}
 &\int_0^\infty \frac{1}{2x^2+3x+1} dx \\
 &= \lim_{t \rightarrow 0} \left[\ln\left(\frac{2t+1}{t+1}\right) \right]_t^1 + \lim_{t \rightarrow \infty} \left[\ln\left(\frac{2t+1}{t+1}\right) \right]_1^t \\
 &= \cancel{\ln\frac{3}{2}} - \ln 1 + \ln 2 - \cancel{\ln\frac{3}{2}} \\
 &= \ln 2
 \end{aligned}$$

Differentiating Inverse Trigonometric Functions

Find $\frac{d}{dx} \sin^{-1} x$

Let $y = \sin^{-1} x$

$$\begin{aligned}
 \sin y &= x \\
 \cos y \frac{dy}{dx} &= 1 \\
 \frac{dy}{dx} &= \frac{1}{\cos y} \\
 \frac{dy}{dx} &= \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}
 \end{aligned}$$

$$\text{Find } \frac{d}{dx} \tan^{-1} x$$

$$\text{Let } y = \tan^{-1} x$$

$$\tan y = x$$

$$\sec^2 y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1 + \tan^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1 + x^2}$$

$$\frac{d}{dx} \cos^{-1} x$$

$$\text{Let } y = \cos^{-1} x$$

$$\cos y = x$$

$$-\sin y \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{-\sin y}$$

$$= -\frac{1}{\sqrt{1 - \cos^2 y}}$$

$$= -\frac{1}{\sqrt{1 - x^2}}$$

$$\frac{d}{dx} x^2 \cos^{-1} x = x^2 \left(-\frac{1}{\sqrt{1-x^2}} \right) + 2x \cos^{-1} x$$

$$= -\frac{x^2}{\sqrt{1-x^2}} + 2x \arccos x$$