Edexcel FP2 Jun 2008 Differential Equations Questions - Mark Scheme

1.	Integrating factor $= e^{-3x}$	B1	
	$\therefore \frac{d}{dx}(ye^{-3x}) = xe^{-3x}$	M1	
	$\therefore (ye^{-3x}) = \int xe^{-3x}dx = -\frac{x}{3}e^{-3x} + \int \frac{1}{3}e^{-3x}dx$	M1	
	$= -\frac{x}{3}e^{-3x} - \frac{1}{9}e^{-3x}(+c)$	A1	
	$\therefore y = -\frac{x}{3} - \frac{1}{9} + ce^{3x}$	A1ft	5

Notes:

First M for multiplying through by Integrating Factor and evidence of calculus

Second M for integrating by parts 'the right way around'.
Be generous – ignore wrong signs and wrong constants.

Second M dependent on first. Both As dependent on this M.

First A1 for correct expression – constant not required

Second A requires constant for follow through.

If treated as a second order de with errors then send to review.

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3.	(a)	Solve auxiliary equation $3m^2 - m - 2 = 0$ to obtain $m = -\frac{2}{3}$ or 1	M1A1	
		C.F is $Ae^{-\frac{2}{3}x} + Be^{x}$	A1ft	
		Let PI $= \lambda x^2 + \mu x + v$. Find $y' = 2\lambda x + \mu$, and $y'' = 2\lambda$ and substitute into i.e.	M1	
		Giving $\lambda = -\frac{1}{2}, \mu = \frac{1}{2}$ and $v = -\frac{7}{4}$,	A1A1A1	
		$\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + Ae^{-\frac{2}{3}x} + Be^{x}$	A1ft	8

Attempt to solve quadratic expression with 3 terms (usual rules)

Both values required for first accuracy.

Real values only for follow through

Second M 3 term quadratic for PI required

Final A1ft for their CF+ their PI dependent upon at least one M

(b) Use boundary conditions:

$2 = -\frac{7}{4} + A + B$ M1A1ft

$y' = -x + \frac{1}{2} - \frac{2}{3}Ae^{-\frac{2}{3}x} + Be^{x}$ and $3 = \frac{1}{2} - \frac{2}{3}A + B$ M1A1

Solve to give $A = 3/4$, $B = 3$ ($\therefore y = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{7}{4} + \frac{3}{4}e^{-\frac{2}{3}x} + 3e^{x}$) M1 A1 6

Second M for attempt to differentiate their y and third M for substitution

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5. (a) $m^2 + 4m + 3 = 0$ $m = -1, m = -3$ M1A1

C.F. $(x =)Ae^{-t} + Be^{-3t}$ must be function of t, not x A1

P.I. $x = pt + q$ (**or** $\boldsymbol{x} = at^2 + bt + c$) B1

$4p + 3(pt + q) = kt + 5$ $3p = k$ (Form at least one eqn. in p and/or q) M1

$4p + 3q = 5$

$p = \frac{k}{3}, q = \frac{5}{3} - \frac{4k}{9}\left(= \frac{15-4k}{9}\right)$ A1

General solution: $x = Ae^{-t} + Be^{-3t} + \frac{kt}{3} + \frac{15-4k}{9}$

must include $x =$ and be function of t A1ft 7

M1 for auxiliary equation substantially correct

B1 not awarded for $x = kt$ + constant

(b) When $k = 6$, $x = 2t - 1$ M1 A1cao

M mark for using $k = 6$ to derive a linear expression in t.
(cf must have involved negative exponentials only)
so e.g. $y = 2t - 1$ is M1 A0

[9]

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7. (a) $\frac{dy}{dx} = v + x\frac{dv}{dx}$ B1

$\left(v + x\frac{dv}{dx}\right) = \frac{x}{vx} + \frac{3vx}{x} \Rightarrow x\frac{dv}{dx} = 2v + \frac{1}{v}$ (*) M1A1 3

B1 for statement printed or for $\frac{dy}{dx} = (x + v\frac{dx}{dv})\frac{dv}{dx}$

First M1 is for RHS of equation only but for A1 need whole answer correct .

(b) $\int \frac{v}{1+2v^2}\,dv = \int \frac{1}{x}dx$ M1

$\frac{1}{4}\ln(1+2v^2), = \ln x(+C)$ dM1A1, B1

$Ax^4 = 1 + 2v^2$ dM1

$Ax^4 = 1 + 2\left(\frac{y}{x}\right)^2$ so $y = \sqrt{\frac{Ax^6 - x^2}{2}}$ or $y = x\sqrt{\frac{Ax^4 - 1}{2}}$

or $y = x\sqrt{\left(\frac{1}{2}e^{4\ln x + 4c} - \frac{1}{2}\right)}$ M1A1 7

First M1 accept $\int \frac{1}{2v + \frac{1}{v}}dv = \int \frac{1}{x}dx$

Second M1 requires an integration of correct form ¼ may be missing

A1 for LHS correct with ¼ and B1 is independent and is for ln*x*

Third M1 is **dependent** and needs correct application of log laws

Fourth M1 is independent and merely requires return *to y/x* for *v*

N.B. There is an IF method possible after suitable rearrangement – see note.

(c) $x = 1$ at $y = 3$: $3 = \sqrt{\frac{A-1}{2}}$ $A = ...$ M1

$y = \sqrt{\frac{19x^6 - x^2}{2}}$ or $y = x\sqrt{\frac{19x^4 - 1}{2}}$ A1 2

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9.	(a)	$(x^2+1)\dfrac{d^3y}{dx^3}+2x\dfrac{d^2y}{dx^2}=4y\dfrac{dy}{dx}+(1-2x)\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}$	M1A1	
		$(x^2+1)\dfrac{d^3y}{dx^3}=(1-4x)\dfrac{d^2y}{dx^2}+(4y-2)\dfrac{dy}{dx}$ (*)	A1	3
		M: Use of product rule (at least once) and implicit differentiation (at least once).		
	(b)	$\left(\dfrac{d^2y}{dx^2}\right)_0=3$	B1	
		$\left(\dfrac{d^3y}{dx^3}\right)_0=5$ Follow through: $\dfrac{d^3y}{dx^3}=\dfrac{d^2y}{dx^2}+2$	B1ft	
		$y=1+x+\dfrac{3}{2}x^2+\dfrac{5}{6}x^3\ldots$	M1A1	4
		M: Use of series expansion with values for the derivatives (can be allowed without the first term 1, and can also be allowed if final term uses 3 rather than 3!)		
	(c)	$x=-0.5,\ y\approx 1-0.5+0.375-0.104166\ldots$ = 0.77 (2 d.p.) [awrt 0.77]	B1	1
			[8]	

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