

(b) Let

$$C = \cos \theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \dots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

$$\text{and } S = \sin \theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \dots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

where  $n$  is an integer greater than 1.

By considering  $C + jS$ , show that  $C = 0$  and  $S = 0$ .

$$\begin{aligned}
 C + jS &= e^{j\theta} + e^{j(\theta + \frac{2\pi}{n})} + e^{j(\theta + \frac{4\pi}{n})} + \dots + e^{j(\theta + \frac{(2n-2)\pi}{n})} \\
 &= e^{j\theta} + e^{j\theta} e^{\frac{j2\pi}{n}} + e^{j\theta} e^{\frac{j4\pi}{n}} + \dots + e^{j\theta} e^{\frac{j(2n-2)\pi}{n}} \\
 \text{n terms} \quad \text{GP} \quad a = e^{j\theta}, \quad r = e^{\frac{j2\pi}{n}} \quad n = n \\
 C + jS &= S_n = \frac{a(1-r^n)}{1-r} = \frac{e^{j\theta}(1 - (e^{\frac{j2\pi}{n}})^n)}{1 - e^{\frac{j2\pi}{n}}} \\
 C + jS &= \frac{e^{j\theta}(1 - e^{j2\pi})}{1 - e^{\frac{j2\pi}{n}}} = \frac{e^{j\theta}(1 - 1)(1 - e^{-j\frac{2\pi}{n}})}{(1 - e^{\frac{j2\pi}{n}})(1 - e^{-j\frac{2\pi}{n}})} \\
 &= \frac{e^{j\theta}(0)(1 - e^{-j\frac{2\pi}{n}})}{1 - (e^{\frac{j2\pi}{n}} + e^{-\frac{j2\pi}{n}}) + 1} \\
 &= \frac{0}{2 - 2 \cos \frac{2\pi}{n}} = 0
 \end{aligned}$$

Eq Re and Im parts

$$C = 0 \quad S = 0$$


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- 2 (a) The infinite series  $C$  and  $S$  are defined as follows.

$$C = a \cos \theta + a^2 \cos 2\theta + a^3 \cos 3\theta + \dots,$$

$$S = a \sin \theta + a^2 \sin 2\theta + a^3 \sin 3\theta + \dots,$$

where  $a$  is a real number and  $|a| < 1$ .

By considering  $C + jS$ , show that

$$S = \frac{a \sin \theta}{1 - 2a \cos \theta + a^2}.$$

Find a corresponding expression for  $C$ . [8]

$$C + iS = ae^{i\theta} + a^2 e^{2i\theta} + a^3 e^{3i\theta} + \dots$$

$$\text{Infinite GP } 'a' = ae^{i\theta}, r = ae^{i\theta}$$

$$C + iS = S_\infty = \frac{a}{1-r} = \frac{ae^{i\theta}}{1 - ae^{i\theta}}$$

$$C + iS = \frac{ae^{i\theta} (1 - ae^{-i\theta})}{(1 - ae^{i\theta})(1 - ae^{-i\theta})}$$

$$= \frac{ae^{i\theta} - a^2}{1 - a(e^{i\theta} + e^{-i\theta}) + a^2}$$

$$= \frac{a(\cos \theta + i \sin \theta) - a^2}{1 + a^2 - 2a \cos \theta}$$

Eq Re and Im parts

$$C = \frac{a \cos \theta - a^2}{1 - 2a \cos \theta + a^2} \quad S = \frac{a \sin \theta}{1 - 2a \cos \theta + a^2}$$


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2 (a) (i) Show that

$$1 + e^{j2\theta} = 2 \cos \theta (\cos \theta + j \sin \theta). \quad [2]$$

(ii) The series  $C$  and  $S$  are defined as follows.

$$C = 1 + \binom{n}{1} \cos 2\theta + \binom{n}{2} \cos 4\theta + \dots + \cos 2n\theta$$

$$S = \binom{n}{1} \sin 2\theta + \binom{n}{2} \sin 4\theta + \dots + \sin 2n\theta$$

By considering  $C + jS$ , show that

$$C = 2^n \cos^n \theta \cos n\theta,$$

and find a corresponding expression for  $S$ . [7]

a)  $1 + e^{j2\theta} = 1 + \cos 2\theta + j \sin 2\theta$   
 $= 1 + 2 \cos^2 \theta - 1 + j 2 \sin \theta \cos \theta$   
 $= 2 \cos \theta (\cos \theta + j \sin \theta)$

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$$\begin{aligned} C + jS &= 1 + \binom{n}{1} e^{j2\theta} + \binom{n}{2} e^{j4\theta} + \dots + \binom{n}{n} e^{j2n\theta} \\ &= (1 + e^{j2\theta})^n \\ &= (2 \cos \theta (\cos \theta + j \sin \theta))^n \\ C + jS &= 2^n \cos^n \theta (\cos n\theta + j \sin n\theta) \end{aligned}$$

Eq Re + Im part

$$C = 2^n \cos^n \theta \cos(n\theta)$$

$$S = 2^n \cos^n \theta \sin(n\theta)$$