We want $\dfrac{a+bi}{c}$ where $a, b, c$ real

GP $\Sigma = \dfrac{a(1-r^n)}{1-r}$

this leads to denominator

$$1 - e^{i3\theta}$$

$$(1-e^{i3\theta})(1-e^{-i3\theta})$$
$$1 - e^{i3\theta} - e^{-i3\theta} + 1$$
$$= 2 - (e^{i3\theta} + e^{-i3\theta})$$
$$= 2 - 2\cos 3\theta$$

---

Another example

Denominator $3 + 2e^{i\theta}$

$$3 + 2\cos\theta + 2i\sin\theta$$

$$\big((3+2\cos\theta) + 2i\sin\theta\big)\big((3+2\cos\theta) - 2i\sin\theta\big)$$
$$= (3+2\cos\theta)^2 + (2\sin\theta)^2$$

$$= 9 + 12\cos\theta + 4\cos^2\theta + 4\sin^2\theta$$

$$= 13 + 12\cos\theta$$

---

Exercise 1E

1 a) $z = e^{\frac{\pi i}{n}}$

Show $1 + z + z^2 + \dots z^{2n-1} = 0$

GP $a = 1$, $r = e^{\frac{\pi i}{n}}$, no of terms $= 2n$

$$S_{2n} = \frac{a(1 - r^{2n})}{1 - r}$$

$$= \frac{1\left(1 - \left(e^{\frac{\pi i}{n}}\right)^{2n}\right)}{1 - e^{\frac{\pi i}{n}}}$$

$$= \frac{1(1 - e^{2\pi i})}{1 - e^{\frac{\pi i}{n}}}$$

$$= \frac{1(1 - (\cos 2\pi + i\sin 2\pi))}{1 - e^{\frac{\pi i}{n}}}$$

$$= \frac{1(1 - 1 - 0)}{1 - e^{\frac{\pi i}{n}}} = 0$$

---

3) $$\sum_{r=0}^{7}(1+i)^r = -15i$$

$$(1+i) = \sqrt{2}\,e^{i\frac{\pi}{4}}$$

GP $\quad a = 1 \quad , \quad r = \sqrt{2}\,e^{i\frac{\pi}{4}} \quad , \quad n = 8$

$$S_8 = \frac{a(1-r^8)}{1-r}$$

$$= \frac{1\left(1-16e^{i2\pi}\right)}{1-\sqrt{2}\,e^{i\frac{\pi}{4}}}$$

$$= \frac{-15}{1-\sqrt{2}\,e^{i\frac{\pi}{4}}}$$

$$= \frac{-15\left(1-\sqrt{2}\,e^{-i\frac{\pi}{4}}\right)}{\left(1-\sqrt{2}\,e^{i\frac{\pi}{4}}\right)\left(1-\sqrt{2}\,e^{-i\frac{\pi}{4}}\right)}$$

$$= \frac{-15+15\sqrt{2}\,e^{-i\frac{\pi}{4}}}{1-\sqrt{2}\left(e^{i\frac{\pi}{4}}+e^{-i\frac{\pi}{4}}\right)+2}$$

$$= \frac{-15+15\sqrt{2}\left(\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)}{3-2\sqrt{2}\cos\frac{\pi}{4}}$$

$$= \quad = -15+15-15i$$

1

$$= -15i$$