

Exercise 1c

$$1a) (\cos\theta + i \sin\theta)^6 = \cos 6\theta + i \sin 6\theta$$

$$1d) \left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)^8 = \cos\frac{8\pi}{3} + i \sin\frac{8\pi}{3}$$

$$= -\frac{1}{2} + i \frac{\sqrt{3}}{2}$$

$$2a) \frac{\cos 5\theta + i \sin 5\theta}{(\cos 2\theta + i \sin 2\theta)^2} = \frac{(\cos\theta + i \sin\theta)^5}{(\cos\theta + i \sin\theta)^4} = \cos\theta + i \sin\theta$$

$$= e^{i\theta}$$

$$3a) \frac{\left(\cos\frac{7\pi}{13} - i \sin\frac{7\pi}{13}\right)^4}{\left(\cos\frac{4\pi}{13} + i \sin\frac{4\pi}{13}\right)^6} = \frac{\left(\cos\left(-\frac{7\pi}{13}\right) + i \sin\left(-\frac{7\pi}{13}\right)\right)^4}{e^{i\left(\frac{24\pi}{13}\right)}}$$

$$= \frac{e^{-i\frac{28\pi}{13}}}{e^{i\frac{24\pi}{13}}}$$

$$= e^{-i\frac{52\pi}{13}} = e^{-i4\pi}$$

$$= \cos(-4\pi) + i \sin(-4\pi)$$

$$= 1$$

$$4a) (1+i)^5$$

$$\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^5$$

$$= 4\sqrt{2}e^{i\frac{5\pi}{4}}$$

$$= 4\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$$

$$= -4 - 4i$$

$$|1+i| = \sqrt{2}$$

$$\arg(1+i) = \frac{\pi}{4}$$

$$4d) (1 - i\sqrt{3})^6$$

$$(2e^{-i\frac{\pi}{3}})^6$$

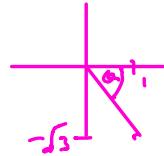
$$= 64 e^{-i2\pi}$$

$$= 64 (\cos(-2\pi) + i \sin(-2\pi))$$

$$= 64$$

$$|1 - i\sqrt{3}| = 2$$

$$\arg(1 - i\sqrt{3}) = -\frac{\pi}{3}$$



$$4e) \left(\frac{3}{2} - \frac{1}{2}i\sqrt{3}\right)^9$$

$$= \left(\sqrt{3}e^{-i\frac{\pi}{2}}\right)^9$$

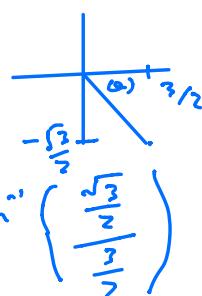
$$= 81\sqrt{3} e^{-i\frac{9\pi}{2}}$$

$$= 81\sqrt{3} e^{i\frac{\pi}{2}}$$

$$= 81\sqrt{3} i$$

$$\left|\frac{3}{2} - \frac{1}{2}i\sqrt{3}\right| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$$

$$\arg\left(\frac{3}{2} - \frac{1}{2}i\sqrt{3}\right) = -\frac{\pi}{2}$$



$$\theta = \tan^{-1}\left(\frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}}\right)$$

$$\theta = \tan^{-1}\frac{1}{\sqrt{3}}$$

$$8a) \frac{1+i\sqrt{3}}{1-i\sqrt{3}} = \frac{2e^{i\frac{\pi}{3}}}{2e^{-i\frac{\pi}{3}}} = e^{i\frac{2\pi}{3}}$$

$$b) (e^{i\frac{2\pi}{3}})^n = e^{i\frac{2\pi n}{3}} \quad n=3 \Rightarrow e^{i2\pi}$$

real and true

$$9) |a+bi| = \sqrt{a^2+b^2} = r \text{ say}$$

$$|a-bi| = \sqrt{a^2+b^2} = r$$

$$\text{Let } \arg(a+bi) = \phi \Rightarrow \arg(a-bi) = -\phi$$

$$(a+bi)^n = r^n e^{in\theta}$$

$$(a-bi)^n = r^n e^{-in\theta}$$

$$(a+bi)^n + (a-bi)^n = r^n \left( e^{in\theta} + e^{-in\theta} \right)$$

$$= r^n (2 \cos n\theta)$$

which is real for all integers  $n$

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## Important Results

$$z + \frac{1}{z} = 2 \cos \theta \quad z^n + \frac{1}{z^n} = 2 \cos n\theta$$

$$z - \frac{1}{z} = 2i \sin \theta \quad z^n - \frac{1}{z^n} = 2i \sin n\theta$$

where  $z = \cos \theta + i \sin \theta$

$$\frac{1}{z} = z^{-1} = \cos \theta - i \sin \theta$$

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### Exercise 1D →

$$\text{1a) } (\cos \theta + i \sin \theta)^3 = \cos 3\theta + i \sin 3\theta$$

$$(c + is)^3 = c^3 + 3c^2is + 3ci^2s^2 + i^3s^3$$

$$= c^3 + 3c^2is - 3cs^2 - is^3$$

Eg real and imaginary parts

$$\begin{aligned} \cos 3\theta &= c^3 - 3cs^2 \\ &= c^3 - 3c(1-c^2) \\ &= c^3 - 3c + 3c^3 \\ &= 4c^3 - 3c \end{aligned}$$

$$\begin{aligned}\cos 3\alpha &= 4\cos^3\alpha - 3\cos\alpha \\ \sin 3\alpha &= 3s^2c - s^3 \\ \sin 3\alpha &= 3(1-s^2)s - s^3 \\ &= 3s - 3s^3 - s^3 \\ &= 3\sin\alpha - 4\sin^3\alpha\end{aligned}$$

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Hwk Finish Q1 of Exercise 1D

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